chapter / 8

EFFICIENT ORGANIZATION WITHIN MARKET AREAS[1]

8.1 EFFICIENT PRODUCT ASSEMBLY

Economical assembly of a product from scattered producing units may well involve a combination of plant and transportation operations. In fact, commodity marketing will frequently involve plant operations, either as an adjunct to the collection operation or for involved processing. The marketing of farm products affords many examples of such plant facilities —packing houses, canneries, cotton gins, country grain elevators, creameries, and cheese factories to mention but a few. We now examine the problem of the economical and efficient organization of plants plus transportation when there are many scattered sources for the raw product or destinations for the finished goods.

Although earlier chapters have dealt with the allocation of producing territories among competing markets, the same general principles apply to

[1]In this chapter the mathematical formulation of plant location model is based on John F. Stollsteimer, "A Working Model for Plant Numbers and Locations," *Journal of Farm Economics*, Vol. 45, No. 3 (August, 1963), pp. 631–645. Empirical data on the sweet potato industry are drawn from Gene A. Mathia and Richard A. King, *Planning Data for the Sweet Potato Industry: 3. Selection of the Optimum Number, Size and Location of Processing Plants in Eastern North Carolina*, North Carolina State College, A. E. Information Series No. 97 (Raleigh, 1962), 75 pp.

the allocation of producing territories among local marketing facilities. Two important adjustments to the analysis are required: (1) the market demand function is replaced by a plant cost curve or, in the long run, by a curve representing economies of scale and (2) the number, size, and location of plants are variables rather than fixed as in the given geographic distribution of cities.

The economical organization of a system that is concerned with processing plants involves the simultaneous consideration of three main components of total cost: (1) the costs of collection from scattered origins to the point of plant location, (2) the costs of plant operation, and (3) the costs of plant-to-market transportation. All of these components can be expected to vary with variations in the total volume handled by the plant, and the most economical organization will involve the selection of that plant volume which will result in minimum *combined* costs for the three component operations. In the following discussion, however, we stress the combination of collection and plant operations. This does not mean that plant-to-market transportation is unimportant; instead, it is omitted for simplicity and because the insertion of this element in the analysis is not difficult.[2]

8.2 ISOLATED PLANT SITES

Consider a plant located in the middle of a large producing territory. This plant is in essence a very small market, and to attract larger and larger volumes it will be necessary to offer higher and higher prices for the product delivered to the plant door. As long as the plant is isolated from other plants, its supply or producing territory will take the form of a circle centered on the plant. If the density of production is held constant, it is clear that the volume delivered to the plant will be a direct function of the circular area and, hence, of the square of the radius of this area (r^2). Collection costs will tend to increase with distance at a constant rate, but because of the quadratic relation between distance and volume the marginal costs of collection will increase with volume at a decreasing rate. Total collection costs, it follows will be related to the cube of the radius (r^3).

The specific relationship between total volume and collection costs will depend on the particular geographic pattern of production, since this pattern will determine the extent of the plant area for any selected volume but, in general, it can be expected to resemble the curves given in Figure 8.1. They illustrate three different situations: (1) relatively high density,

[2]Miller and King (1964) discuss a variety of models which are appropriate for more complicated situations.

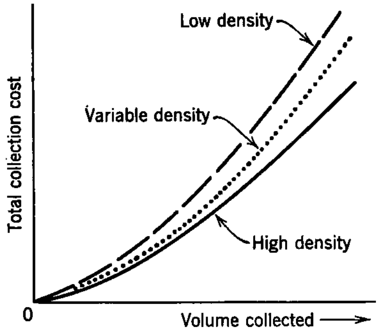

FIGURE 8.1 The effect of volume and production density on total collection costs for an expanding plant area.

constant throughout the producing territory, (2) constant density but at a lower level (actually, we have illustrated the case where density is one-half that in the first case), and (3) variable density, graduated from high density near the plant location to low density in distant sections. In all cases, the relationship indicates that total collection costs increase with total volume at an increasing rate: the rate increase is more rapid as density is lowered, reflecting the proportionately greater distances involved in obtaining a fixed increment to plant volume.

If there were no economies of scale in plant operation—if plant costs per unit of product were constant and unaffected by plant size—then, clearly, optimum organization would involve a plant at every production location. But plants are subject to economies—and perhaps diseconomies—of scale; at least within limits, larger plants with a volume well adjusted to the available capacity will operate with lower average costs than smaller plants. Optimum organization, therefore, involves a balancing of the decreasing average plant costs against the increasing collection costs. This is suggested by the diagram in Figure 8.2 where we show the total long-run cost and volume relationship for plant operation in conventional sigmoid form. This relationship suggests that unit costs for plant operation will decrease over a considerable range in scale and output and eventually increase. When combined with the relationship for collection costs, we can indicate the point *b* that represents the lowest possible costs per unit for the collection plus plant operations.

This combination of plant and collection costs is also shown in terms of average costs in Figure 8.3. Notice that plant costs alone are at lowest levels at point *a* and that combined costs are minimized at a lower volume as represented by point *b*. A consideration of the several diagrams should

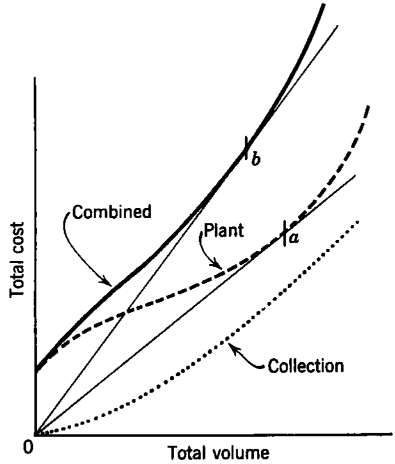

FIGURE 8.2 The combination of collection costs and long-run plant costs to determine optimum organization.

indicate that minimum combined costs will involve lower volumes as density is decreased, although the reduction in volume with lower densities will usually involve expansions in the geographic area served by the plant. In this isolated situation, in any event, optimum organization will involve balancing off plant and collection costs; and the final adjustment will entail a plant located in the center of a circular supply area.[3]

[3]An interesting empirical example is provided by Henry, Chappell and Seagraves (1960).

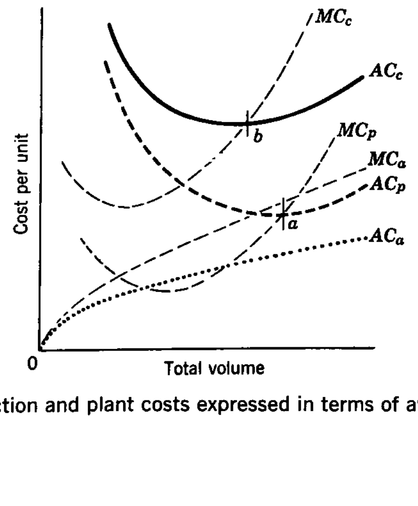

FIGURE 8.3 Collection and plant costs expressed in terms of average costs per unit of product handled.

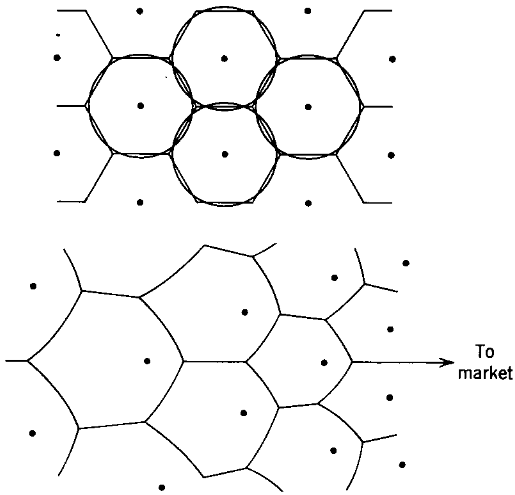

FIGURE 8.4 A regular Lösch system of hexagonal market areas, and modifications to represent supply areas around country marketing plants.

8.3 COMPETING PLANTS

With the development of competing plants to service an entire producing region, free-choice areas would allocate the region among plants much as in the case of competing markets; but now the final equilibrium would determine the location, size, and number of plants as well as the allocation of territory among them. Since a system of circular areas cannot completely cover a region without overlapping and since such overlapping would be eliminated by producers in making their free choice of most favorable outlets, the idealized solution to the problem of plant size and location would appear to involve a regular system of hexagonal plant areas as shown in the upper half of Figure 8.4. But plants located at varying distances from the central market will necessarily have differing at-plant prices. This would distort the interplant boundaries somewhat as suggested in Figures 8.5 and 8.6. Further deviations from the regular hexagonal pattern would result from variations in production densities and from the peculiarities of local road networks. With these complications and with the infinite set of alternative volume-number-location possibilities, a rigorous theoretical solution to this problem may well be impossible. Practical and at least near-optimum solutions can be obtained, however, by determining the cost-minimizing volumes appropriate for

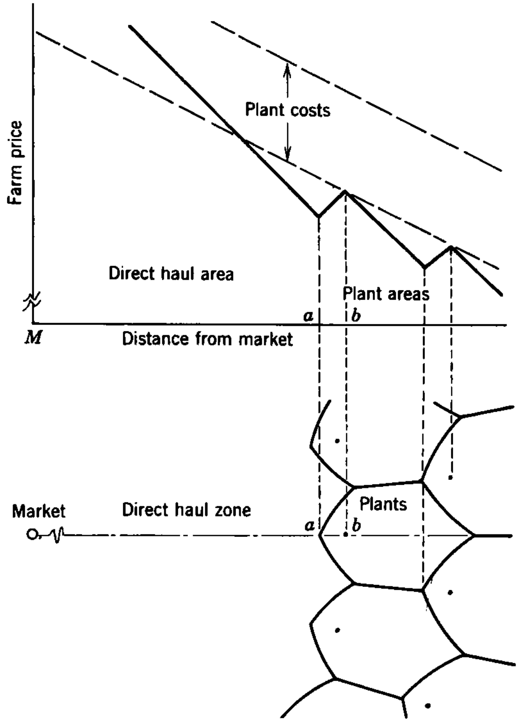

FIGURE 8.5 A cross-section of price surface in a market area with direct haul and plant-plus-transportation zones.

FIGURE 8.6 The allocation of direct haul and plant areas within a market area.

isolated plants in various parts of the market territory and then by using these results as guides in establishing a somewhat arbitrary pattern of plant locations and the associated systems of plant areas and volumes for the entire region.

8.4 PROCESSING PLANT NUMBERS AND COSTS

An analysis of the sweet potato processing industry in Eastern North Carolina will be used to illustrate how the number, size, and location of processing plants can be selected that will minimize combined assembly and processing costs. The production density pattern is regarded as predetermined. The assembly costs would be expected to decline as the number of plants increases because the size of the supply area for particular plants and total distance required for assembly are reduced. This effect is illustrated in Figure 8.7 where *TAC* represents the minimum total assembly costs associated with assembling a fixed volume of raw product with varying numbers of plants.

The effect of the number of processing plants on total season processing costs, *TSPC* is illustrated in Figure 8.8. The *TSPC* curve should not be confused with the usual total cost function where cost is expressed as a function of *plant* volume. A positive sloping *TSPC* function reflects higher costs that are associated with processing a given volume of raw material in two or more plants as compared with the processing of the same volume in any one plant. Two plants would each process one-half the volume handled by a one plant organization of the region. The slope of *TSPC*, thus, reflects the added annual costs of building and operating the processing sector as the number of plants is increased from j to $j+1$. This assumes that plants can be designed for any output and that they will be operated at a specified proportion (100 percent?) of capacity.

The summation of these two relationships results in a combined assembly and processing function that is useful in evaluating the efficiency of the marketing system. The procedure for summing the assembly and processing relationships is described in a mathematical note at the end of this chapter. The combined cost function *TC* and a hypothetical two-plant optimum solution are shown in Figure 8.8. The optimum number and location of plants are determined when the reduction in assembly costs is just offset by the increase in processing costs as the number of plants increases.

The total assembly costs are minimized for a given set of plants by

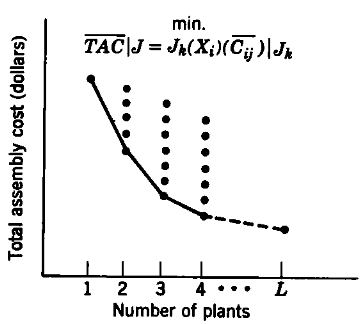

FIGURE 8.7 The minimized total assembly costs for fixed volume of raw product. (See Section 8.7 for details and definitions of terms.)

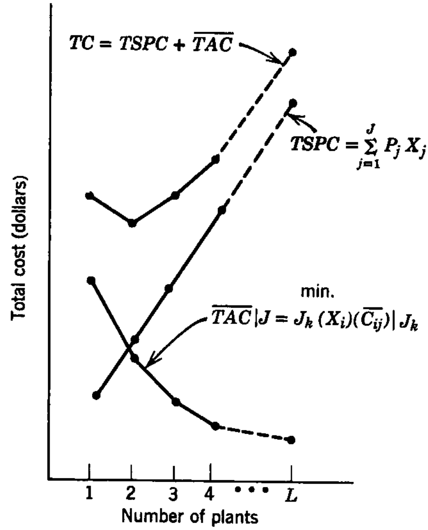

FIGURE 8.8 The minimized total assembly and the processing plant cost for a fixed volume of raw product. (See Section 8.7 for details and definitions of terms.)

assigning the fixed supplies of raw potatoes at each production origin to that plant which minimizes assembly costs from that origin. This is possible because of the assumption of equal FOB plant prices (see Mathia and King, 1962). The assembly cost-distance relationship in terms of road miles for a two-ton truck is estimated as follows:

$$C = \$5.14 + \$.176D_R \tag{8.1}$$

where C is total assembly costs in dollars per load and D_R is distance in road miles, round trip.

The important variables affecting total season processing cost are rate of plant operation, length of operating season, percent trim and peel loss, percent of rated capacity, and length of planning horizon. On the assumption of full capacity operations and a 10-year planning horizon, the processing cost equation fitted by least-squares regression to the data for the four model plants described by Hammond (1961) is as follows:

$$\begin{aligned} TSPC = {} & 30{,}560.00 + 26.73H + 173.50R + \\ & 226.50T + 2.324HR + .02662HRT \end{aligned} \tag{8.2}$$

where $TSPC$ = total season processing costs
H = hours of operation per season
R = rate of operation in cases per hour
T = percent trim and peel loss.

The procedure used for determining the optimum rates of output and lengths of season for varying volumes processed is shown in the mathematical note. The linear total cost equation fitted by least-squares regression to estimated $TSPC$ for varying rates of output and lengths of season with trim and peel loss of 40 percent is

$$TSPC^* = \$82{,}781 + \$3.4906V \tag{8.3}$$

where $TSPC^*$ is minimum total season processing costs and V is volume of output in cases per season. The intercept value of $82,781 represents an estimate of the minimum annual cost of establishing and maintaining a processing plant. This minimum value is used in determining the number of plants required to minimize combined assembly and processing costs.

To estimate the minimum costs of assembling sweet potatoes with varying numbers of plants is a sizable computational task. If 22 possible plant sites are considered and one site is to be selected, the minimum assembly cost is selected by assigning the total volume to each of the 22 plants in succession and by selecting the plant site representing the minimum costs. When considering two or more plants, all possible combinations of the given number of plants must be compared by assigning the production at each supply origin among plants so as to obtain minimum total assembly costs.

The assumption of constant and equal marginal costs of processing in all plants simplifies the problem of deriving a processing cost relationship with respect to plant numbers. This assumption concerning marginal costs makes it possible to state the relationship between the total season processing costs and the number of plants as follows:

$$TSPC = bV + aN \tag{8.4}$$

where $TSPC$ = total season processing costs
b = marginal cost per case of processing sweet potatoes
V = total volume to be processed in cases per season
a = minimum season cost of establishing and maintaining a processing plant
N = number of plants in operation.

Appropriate values of V and N were substituted in Equation 8.4 to derive each $TSPC$ by using the parameters of Equation 8.3.

The final step in determining the optimum number, size, and location of plants involves estimating total assembly and processing costs for varying numbers of plants. The relationship between the combined cost and the numbers of sweet potato processing plants in eastern North Carolina is presented in Table 8.1. Notice that one plant constitutes the optimum.

TABLE 8.1 The Relationship Between Number of Plants, Total Assembly Cost, Total Season Processing Cost, Total Combined Cost, and Average Combined Cost

Number of Plants	Total Assembly Cost	Total Season Processing Cost	Total Combined Cost	Average Combined Cost
	1,000 Dollars per Season			Dollars per Case
1	109.0	1,987.5	2,096.5	3.842
2	77.3	2,070.3	2,147.6	3.936
3	58.4	2,153.1	2,211.5	4.053
4	53.2	2,235.8	2,289.0	4.195

Since all possible plant sites were considered to have equal processing-input prices, including the price of green sweet potatoes, assembly costs and the production density pattern are the only factors that determine the optimum locations of plants. The location of plants is automatically specified in the process of determining the optimum number of plants.

The optimum location for one plant was Faison, which was the location with the lowest combined costs of $3.84 per case. Optimum location of plants in the 2-, 3-, and 4-plant industries increased the average cost by 9 cents, 27 cents, and 35 cents, respectively. The raw product supply areas and isotims that indicate the net farm prices associated with the four situations are shown in Figure 8.9*a* to *d*.

8.5 EFFICIENT DISTRIBUTION SYSTEMS

Similar methods can be used to evaluate the efficiency of distribution systems. A 1942 study undertaken to evaluate potential savings of tires, gasoline, and labor required by 55 dealers distributing milk in the metropolitan area of New Haven, Connecticut, indicated that alternate-day

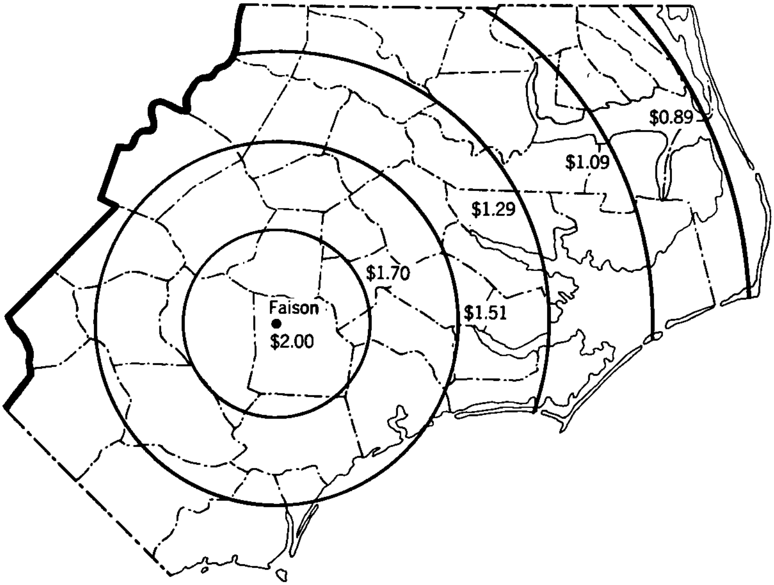

FIGURE 8.9*a* The net farm prices received by producers and the boundary of the supply area for the one best plant located at Faison, North Carolina, 1960.

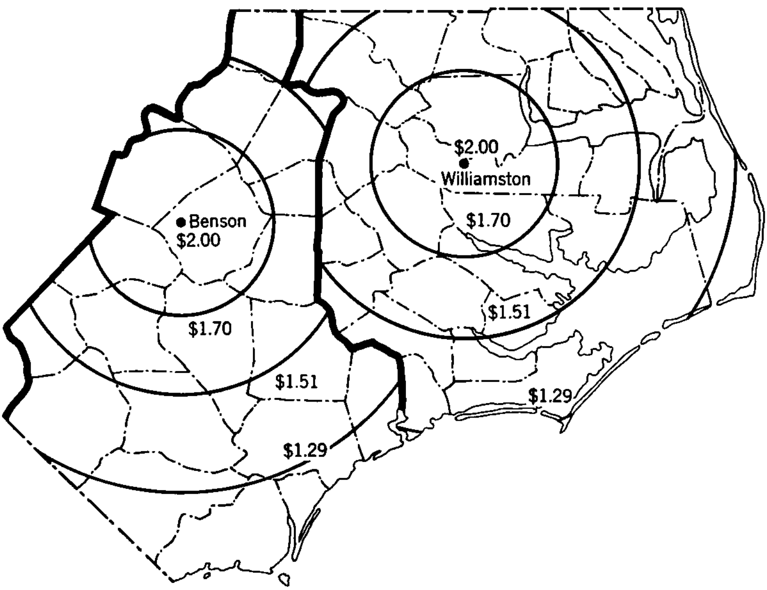

FIGURE 8.9*b* The net farm prices received by producers and the supply area boundaries for the best two plants located at Benson and Williamston, North Carolina, 1960.

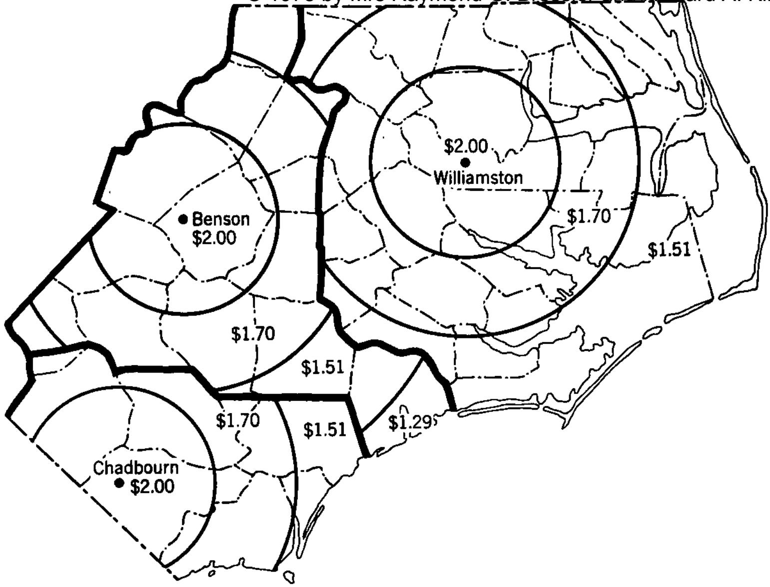

FIGURE 8.9*c* The net farm prices received by producers and the supply area boundaries for the best three plants located at Benson, Chadbourn, and Williamston, North Carolina, 1960.

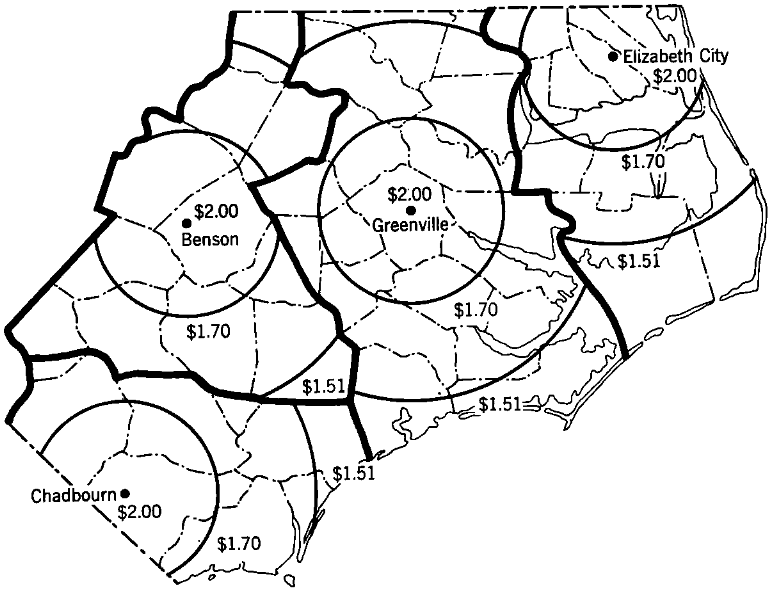

FIGURE 8.9*d* The net farm prices received by producers and the supply area boundaries for the best four plants located at Benson, Chadbourn, Greenville, and Elizabeth City, North Carolina, 1960.

delivery involved 72,936 quarts of milk, 217 routes, and a total route distance of 5284 miles per day. Customers were charged a uniform price regardless of location, and routes duplicated and overlapped to a considerable extent—the 5284 miles of truck travel involved only 700 miles of city streets. If existing dealer plant locations and volumes were maintained but deliveries were allocated through a system of efficient and exclusive territories, the resulting reorganization of routes would have accomplished the total delivery function with 123 routes and 829 miles of daily route travel. Estimated delivery costs could have been reduced from approximately 4.4 to 2.5 cents per quart, a savings of 42 percent representing in the aggregate more than $500,000 per year. This situation in New Haven, it should be emphasized, is by no means abnormal in the field of city milk distribution.

Savings in delivery costs will be maximized if all dealers in a market are involved, yet significant savings are available to smaller subgroups. In New Haven, three small dealers located in different sections of the area were studied. The delivery operations of these three firms were duplicated only in part because their small size and location had resulted in a degree of natural segregation. An exchange that involved 30 percent of all customers would have eliminated this duplication, and with route reorganization truck travel could have been reduced from 242 to 134 miles per day. Delivery costs could have been reduced from an average of 4.5 cents to 3.5 cents per quart—a savings of 22 percent. This would amount to nearly $12,000 per year, an impressive sum for three small firms handling an aggregate of only 3183 quarts of milk daily. Maps that show the actual and theoretical delivery territories are given in Figure 8.10.

Such duplication in distribution routes reflects imperfections in the competitive market structure and, especially, the segmentation of decisions where many firms are involved. Within a particular firm, however, principles of efficient organization may be followed much more precisely. Although routes of all dealers in a milk market overlap and duplicate, the routes operated by a single dealer are normally planned quite rationally and with minimum duplication. In a similar way, routes assemblying or collecting products from farms will usually involve important inefficiencies in duplication and less than optimum size, yet the routes operated by a particular firm will show compact and nonoverlapping areas within the confines imposed by the geographic scatter of the farms selling to this firm. In many cases, then, individual firms may be well adjusted to approximate cost-minimizing allocations, and yet the aggregate situation for the entire market may be quite inefficient. Stated in another way, the principles of location theory and competitive market or supply areas

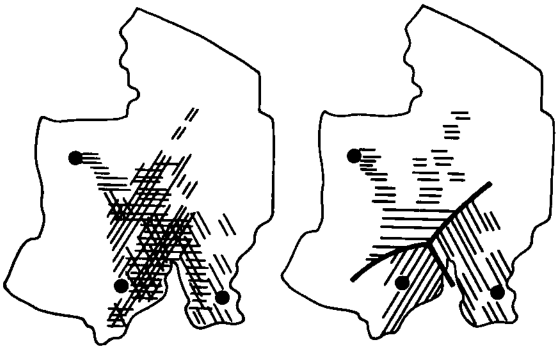

FIGURE 8.10 The actual and reorganized market areas for three small milk distributors in New Haven, Connecticut, 1942. [*Source.* R. G. Bressler, Jr., *City Milk Distribution* ("Harvard Economic Studies," Vol. XCI; Cambridge: Harvard University Press, 1952), p. 278.]

frequently will fail to describe conditions in actual markets, but they would be good descriptions if the entire market operation were placed under a single agency.

8.6 SPATIAL MONOPOLY

Our theoretical models have been based on competitive assumptions, yet we have concluded that efficient organizations of market areas, collection and delivery routes, and local plants all require the allocation of exclusive territories. With such exclusive territories, the number of buyers (or sellers) will be strictly limited, and the local market structure certainly will depart seriously from the competitive assumptions. In short, there is an inherent and unavoidable conflict between competition and efficiency that stems from the element of spatial monopoly. To have even a very limited number of buyers available at every location must mean duplication with unnecessarily high costs for all operators. However, the assigning of exclusive territories to obtain efficiency leaves customers confronted by a single operator and, hence, subject, to some degree, to monopolistic exploitation.

That this is not unrealistic is suggested by the frequent use of devices to absorb freight and hauling charges whereby operators discriminate against nearby producers and subsidize the ones located at a distance from plant. If farm-to-plant transportation is under the control of the plant manager, this may take the form of a flat charge for transportation to all producers regardless of location. Or the competition for producers along the boundaries between plants may actually result in an inverted price structure: distant producers receive high prices in order to gain their trade, while nearby producers with limited alternative outlets receive lower prices.

Spatial monopoly also may be a major element leading to the development of inefficient instead of efficient organizations. Let us suppose that we have an industry organized in a regular and efficient system of hexagonal plant areas where the number and size of areas is consistent with the requirement of minimizing combined plant and transport costs. Such a grid is given in Figure 8.11. Consider the three plants located at *A*, *B*, and *C*; prices for the product *delivered* to these plants will be equal and, consequently, at-farm prices will be lowest at intermediate points such as *D*. Now, if a new operator were to consider entering this field, he might reasonably choose this low-price point for his location. Such actions by a number of newcomers would result in reallocating the area into triangular plant areas with twice the original number of plants. Moreover, low prices at points such as *E* would encourage the continuation of entry, resulting in small hexagonal areas that involved plants with one-third the original volume.

Apparently this process could continue until finally the costs would be so high that a single operator (perhaps a producer cooperative) would find it advantageous to buy out all plants in a section and reestablish the original, cost-minimizing organization. The important issue here is not the

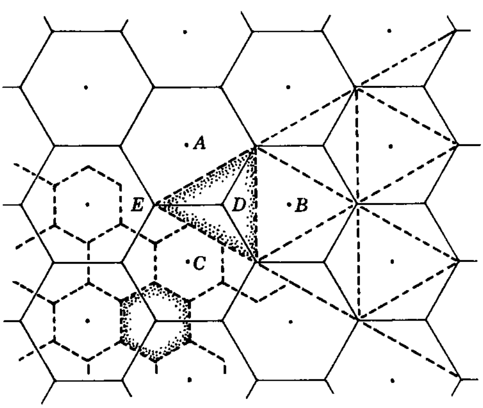

FIGURE 8.11 The degeneration of efficient plant organization as a consequence of the nature of spatial competition.

particular developments that might be involved in this decomposition, but rather that the nature of competition is such that the entry of an excessive number of firms is possible. These firms force inefficiency and high costs on the original plants and, hence, restrict their abilities to compete. When new plants enter, they reduce volumes and increase unit costs for existing plants. Similarly, the overlapping and duplication of collection routes results in excess capacity for all operators in lowered effective collection density and, thus, in higher than optimum costs. This tendency for efficient arrangements to decompose has sometimes been called the "law of mediocracy": because of spatial aspects, competition results in a uniform but unnecessarily high level of costs for all firms instead of the uniform level of low costs that could be achieved through efficient organization. Under these circumstances, location theory can be used to devise efficient organizations, but apparently the implementation and maintenance of them must require direct control by government (public ownership or public utility) or complete monopoly of the entire local marketing function by a single firm.

8.7 MATHEMATICAL NOTE ON EFFICIENT SPATIAL ALLOCATION

The analytical procedure for determining the number, size, and location of processing plants that minimize the combined assembly and processing costs requires statements of the relationships of these two functions to volume of output. The following model adapts the economic logic of location theory to the empirical analysis required in determining the optimum number, size, and location of processing plants.

Given I raw material origins each of which produces a specified quantity X_i of the raw material to be assembled and processed at one of L possible processing plant locations in the supply area, what are the number, size, and location of plants that will minimize the costs of assembling and processing the total quantity of raw material produced in the supply area?

Let

TC = total processing and assembly costs
TAC = total assembly cost
$TSPC$ = total season processing costs
L_j = location of plant j
P_j = unit processing cost of plant j
$(j = 1, \ldots, J \leq L)$ located at L_j
X_j = quantity of raw material processed at plant j

X_i = quantity of raw material produced at origin i per production period
X = total quantity of raw material produced in supply area
X_{ij} = quantity of raw material transported from origin i to plant j located at L_j
C_{ij} = unit cost of transporting the raw product from origin i to plant j located at L_j and
J_k = one combination of locations for J plants among the $\binom{L}{J}$ possible combinations of locations for J plants, given L potential plant locations.

The assembly cost relationship is stated algebraically as

$$\underset{(J,J_k)}{TAC} = \sum_{i=1}^{I} \sum_{j=1}^{J} X_{ij}C_{ij}|J_k. \tag{8.5}$$

The total season processing-cost relationship is expressed as

$$\underset{(J,J_k)}{TSPC} = \sum_{j=1}^{J} P_jX_j|J_k \tag{8.6}$$

These two relationships are stated algebraically as a sum. The procedure is to minimize the combined function

$$\underset{(J,J_k)}{TC} = \sum_{j=1}^{J} P_jX_j\Big|J_k + \sum_{i=1}^{I}\sum_{j=1}^{J} X_{ij}C_{ij}\Big|J_k \tag{8.7}$$

with respect to plant numbers J ($J \leqslant L$) and locations J_k $\left[k = 1, \ldots, \binom{L}{J}\right]$ subject to

$$\sum_{j=1}^{J} X_{ij} = X_i \tag{8.8}$$

$$\sum_{i=1}^{I} X_{ij} = X_j \tag{8.9}$$

$$\sum_{i=1}^{I} X_i = \sum_{j=1}^{J} X_j = X \tag{8.10}$$

$$X_{ij} \geqslant 0 \tag{8.11}$$

$$X_j \geqslant 0. \tag{8.12}$$

The procedure followed in minimizing Equation 8.7 with respect to plant numbers and locations is affected by the presence or absence of economies of scale in processing and the effects of plant location on processing costs. Four possible cases of processing relationships and their effects on the optimum number, size, and location of processing plants are discussed by Stollsteimer (1961). For the purpose of this example, it is assumed that (1) economies of scale in processing exist and (2) processing costs are independent of plant location.

The total plant cost function is assumed to be linear and positively sloping with a positive intercept, and plants at each possible location use the same production techniques. These limiting assumptions simplify the analytical procedure. French et al. (1956), showed that these assumptions are consistent with the cost-output relationship found in many processing operations. It is assumed that this type of relationship is satisfactory for long-run planning purposes.

Under these circumstances, the problem of minimizing Equation 8.7 with respect to plant numbers (J) and locations (J_k) is a three-step process. The first step is to compute a transfer cost function that is minimized with respect to plant locations for varying values of J. For a specified number of plants J there are $\binom{L}{J}$ possible combinations of locations.

The transportation cost table or matrix C_{ij} can be partitioned into a submatrix $(C_{ij}^{*})|J_k$ for each combination of locations J_k. This submatrix $(C_{ij}^{*})|J_k$ will be (IxJ), with the elements of the J columns representing the assembly costs from each origin to one of the plant locations being considered. A $(I \times 1)$ column vector $(\overline{C}_{ij}|J_k)$ is obtained by selecting the minimum c_{ij} from each row of the submatrix $(C_{ij}^{*}|J_k)$. Minimum total transfer cost with J plants at a specified combination of locations J_k is the product of the row vector (X_i) whose elements X_i represent the quantities of raw material available at each of the I origins and the column vector $(\overline{C}_{ij}|J_k)$. For each value of J there are $\binom{L}{J}$ values of $(X_i)'(\overline{C}_{ij}|J_k)$. The minimum of these values over $_k$ is the minimum point on an assembly cost function minimized with respect to plant location. This takes on the functional form of

$$\overline{TAC}|J = \overset{\text{min.}}{J_k}(X_i)'(\overline{C}_{ij}|J_k) \tag{8.13}$$

where $\overline{TAC}$ = total assembly cost minimized with respect to plant locations for each value of J ($J = 1, \ldots, L$)

(X_i) = a $(1 \times I)$ vector whose entries X_i represent the raw material produced at each of the I origins

J_k = a particular combination of J plants

$k = 1, 2, \ldots, \binom{L}{J}$

and $(\overline{C}_{ij}|J_k) =$ a vector whose entries c_{ij} represent minimum transfer cost between each origin and a specified combination of locations J_k for J plants.

The second step in determining the optimum number, size, and location of processing plants is to derive the relationship between the cost of processing a fixed quantity of raw material and varying numbers of plants. With constant and equal marginal processing costs for all plants and a positive intercept in the plant cost function, the total cost of processing a fixed quantity of raw material X will increase by an amount equal to the intercept value of the plant cost function with each additional plant.

The intercept value of the plant cost function does not necessarily represent only the cost of durable goods associated with short-run fixed costs. A better description of this intercept value is that it represents the minimum costs associated with establishing and maintaining a processing plant.

The final step in this three-step process is to derive a total combined cost function by adding the total assembly cost function $\overline{TAC}$ and the total processing cost function $TSPC$ for varying numbers of plants. The number of plants which minimizes this total cost function (TC) depends on the rates of change in $\overline{TAC}$ and $TSPC$. Since the $\overline{TAC}$ function would be expected to decline and $TSPC$ to increase as the number of plants increases, TC would fall only if the decrease in $\overline{TAC}$ was greater than the increase in $TSPC$. Figures 8.7 and 8.8 present an example of the expected shapes of the three functions and illustrate a two-plant optimum solution. The amount processed in each plant is equal to $\sum_{i=1}^{I} X_{ij} = X_j$ for each value of J.

The procedure for estimating the optimum rates of output and lengths of season for varying levels of output (V) to minimize processing costs is a constrained-minimization problem. Total processing costs are minimized subject to the constraint that rate of output (R) times length of season (H) equals V.

The procedure to determine the optimum rates of output and lengths of season can be demonstrated for the total season processing cost function, $TSPC = a + b_1R + b_2H + b_3HR$, where a, b_1, b_2, and b_3 are parameters.

First, form the function

$$Z = a + b_1R + b_2H + b_3HR + \lambda\ (V - RH) \tag{8.14}$$

where λ is an undetermined Lagrange multiplier.

Second, set the partial derivatives of Z with respect to H, R, and λ equal to zero and solve for the three unknowns simultaneously.

$$\frac{\partial Z}{\partial R} = b_1 + b_3 H - \lambda H = 0 \tag{8.15}$$

$$\frac{\partial Z}{\partial H} = b_2 + b_3 R - \lambda R = 0 \tag{8.16}$$

$$\frac{\partial Z}{\partial \lambda} = V - RH = 0. \tag{8.17}$$

Solve Equations 8.15 and 8.16 for λ

$$\lambda = b_3 + \frac{b_1}{H} = b_3 + \frac{b_2}{R}. \tag{8.18}$$

Solve Equation 8.18 for H

$$H = \frac{b_1 R}{b_2}. \tag{8.19}$$

Solve Equation 8.17 for R

$$R = \frac{V}{H}. \tag{8.20}$$

Substitute Equation 8.20 for R in Equation 8.19 and solve for H

$$H = \sqrt{\frac{b_1 V}{b_2}}. \tag{8.21}$$

Substitute Equation 8.21 for H in Equation 8.20 and solve for R

$$R = \sqrt{\frac{b_2 V}{b_1}}. \tag{8.22}$$

For any predetermined level of output V, total season processing costs are minimized when length of season H and rate of output R are given by Equations 8.21 and 8.22.

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