chapter / 5

PRICE EQUILIBRIUM IN SPATIALLY SEPARATED MARKETS

In the previous chapter we discussed the determination of price for a particular commodity as a result of the equilibrium of supply and demand in single and multiple price markets. In the present chapter we deal specifically with *prices* for a single commodity in several spatially separated markets and the *flows* of a single commodity in intermarket or interregional trade. Initially, we assume that transfer costs between markets are unimportant; then, that they are given. (Factors that determine transfer costs will be treated in detail in Chapter 6.)

The usage of the terms "market" and "region" may be puzzling. Region is used to refer to a readily identified geographic area. Whether such an area consists of one market, several markets, or is part of a larger, perhaps nationwide, market is an economic matter, not a geographical issue. Furthermore, although a region may be properly regarded as a market for one commodity, it may not be so regarded for others. When correctly described, the geographical features of a market will be clearly stated, so that there should be no cause for confusion.

5.1 THE TWO-REGION CASE

Consider first the simple case involving a single product produced and consumed in two regions. Supply and demand curves are given for each and, in the absence of trade between them, these curves determine the price for the commodity in each region. This is illustrated by the first two sections of Figure 5.1. In region X, the demand curve D_x and supply curve S_x intersect, resulting in a competitive price represented by oa. In region Y, on the other hand, the somewhat lower demand curve D_y and higher supply curve S_y result in the lower equilibrium price ob. In the absence of commercial contact between the two regions, these prices and the accompanying quantities produced and consumed would represent equilibrium conditions.

But suppose that traders from region Y make contact with region X. They discover that the price for this commodity is lower in Y than in X and that, ignoring transfer costs, they can buy in Y and sell in X at a profit. Traders will engage in such *arbitrage*, therefore, to their own profit. As part of the supply available in Y is transferred to X, however, the price in X will decline while that in Y increases. With the assumption of zero transfer cost, it follows that *arbitrage* will continue so long as the price in X exceeds that in Y. Eventually, the flow of the commodity from region Y to region X will be just large enough to result in the equalization of product prices and the establishment of a single market.

The opening of trade between regions has the effect of bringing the combined demand of the regions to bear on the combined supply conditions. This is illustrated by the third section of Figure 5.1, where the combined supply and demand curves for the two regions are shown. These curves have been added horizontally, combining the quantities that would

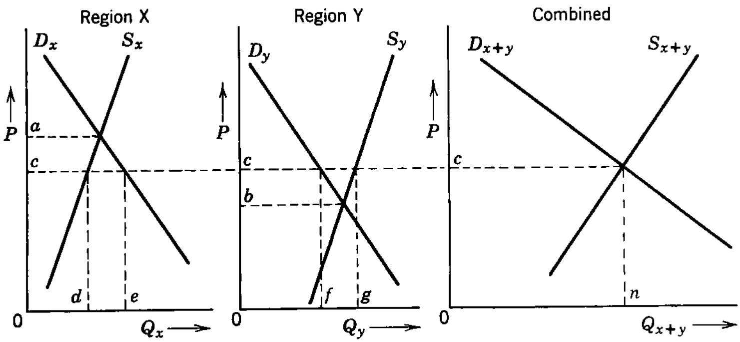


FIGURE 5.1 The trade between two regions as a result of differences in supply and demand functions.

be demanded (or supplied) in the two regions in response to selected levels of price. The intersection of the combined curves indicates a final equilibrium price of *oc* and total output *n*. Notice that this intersection will always occur between the prices that held in the regions in isolation and that it involves shipment of the commodity from the low-price region *Y* to the high-price region *X*. The amount shipped from *Y* is represented by *fg*, and the amount received in region *X* is shown by *de*. These two quantities must be equal, of course, at the equilibrium price *oc*.

An alternative and convenient presentation of the single-product, two-region case is the "back-to-back" diagram of Figure 5.2. Here, the supply and demand curves for region *Y* in Figure 5.1 are plotted on the right half of the diagram in conventional form, but the supply and demand curves for region *X* have been reversed on the left half of the figure: quantities are measured to the right of the origin *O* for region *Y* but to the left for region *X*. Now, suppose we plot an *excess supply* curve for each region, showing the amount by which the quantity offered for sale exceeds the quantity purchased or demanded at various levels of price. They are illustrated by the curves ES_x and ES_y, and their intersection at *j* represents the determination of the equilibrium price *oc* with trade. The distance *cj*, equal to *oh*, represents the quantity of the commodity exported from *Y* to *X*, and this is exactly equal to the *de* and *fg* quantities in this and in the previous diagram.

Observe that interregional trade will expand output in region *Y*, where supply conditions are more favorable, and contract output in region *X*. Also, the higher prices that will prevail in *Y* will reduce local or domestic consumption, while lower prices in *X* will expand consumption. It follows

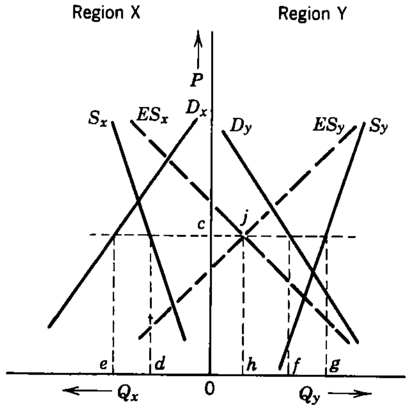


FIGURE 5.2 The equilibrium prices and trade illustrated by a "back-to-back" diagram.

that producers in region *X* will divert some of their productive resources into other uses and that those in region *Y* will allocate more resources to this commodity. On the other hand, consumers in region *X* will purchase more of this commodity and less of others, while consumers in region *Y* will make the opposite adjustment. Clearly, some resistance to such changes may arise in the real world as producer groups fight to protect their home market from producers in other regions. A full analysis of these effects must be delayed until two-commodity models are developed, but we can sense the general problem that new trading opportunities might create.

5.2 INTRODUCTION OF TRANSFER COSTS

We now indicate the modification that the insertion of transfer costs requires in the simple case of trade between two regions in a single product. We have discussed the aggregation of regional supply and demand curves to determine price and quantity traded under the assumption that the transfer costs were zero. We have observed that the two regions would have different prices in the absence of trade and that this difference would give rise to trade flows from region *Y*, where prices were low, to region *X*, where prices were high. This process would continue until prices in the two regions were equalized.

Obviously, this is an oversimplification—there are positive costs involved in the transfer of a commodity from one region to the other. It follows that trade will not completely equalize commodity prices; instead, the prices in the two regions will move toward each other until they differ exactly by the cost of transfer. It is a simple matter to indicate the modifications to the previous analysis that are required by the insertion of transfer costs.

In Figure 5.1, we used the horizontal summation of regional supply curves and demand curves to obtain aggregate functions for the combined regions. This procedure was appropriate in the absence of transfer costs because prices with trade would be identical in the several regions. With the addition of transfer costs, however, equilibrium prices will be lower in the exporting regions than in the importing regions (the difference equaling the transfer cost). In short, supply curves and demand curves can no longer be added directly but must be "positioned" or displaced to reflect this cost. This is indicated in Figure 5.3 where the supply and demand curves for region *Y*—the exporting region—have been moved upward by an amount t representing the unit cost of interregional transfer. Notice that, with this construction, any horizontal line across the diagrams no

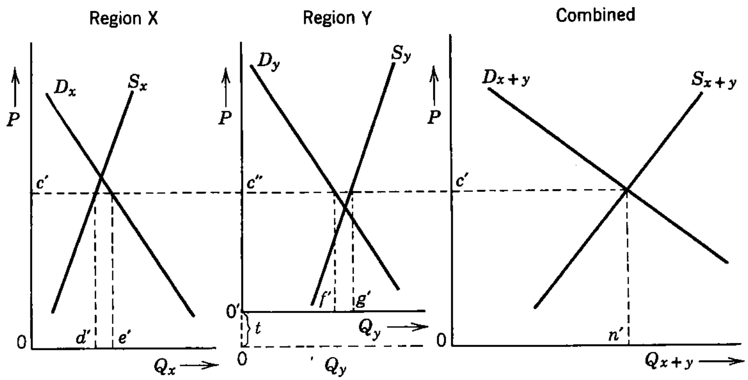


FIGURE 5.3 The effects of transfer cost (t) on prices and trade (compare with Figure 5.1).

longer represents equal prices in the two regions but, instead, prices that differ by transfer costs. Notice also that the combined curves in the final section of the diagram have been expressed in terms of prices in region X, but it is understood that the contributions of region Y are included at the lower level of prices which prevails in that region.

A similar modification can be made to Figure 5.2. Supply and demand curves for region Y, the region with lower prices, can be displaced upward by an amount t which represents the cost of interregional transfer (Figure 5.4). With this displacement, any horizontal line on the back-to-back diagram represents prices in the two regions that differ exactly by transfer cost. The analysis now proceeds as before: excess supply curves are constructed, and their intersection at j' defines the equilibrium prices with trade, equal to oc' in X and $o'c'$ in Y which differ by oo' or t. The distance $c'j'$ now represents the volume traded, and this is equal to the quantity $f'g'$ shipped by Y and $e'd'$ received by X. By simple extension of this argument, it should be clear that trade will take place between two regions only if the prices in isolation differ by more than transfer costs, and that prices in one region can differ from the ones in another by any amount within the range of plus or minus transfer costs without giving rise to commodity movement.

A comparison of Figures 5.2 and 5.4 indicates the general effects of transfer costs on trade. As stated above, commodity prices move toward equality, but equilibrium is reached when prices differ exactly by transfer costs (to go beyond this point would involve losses by the traders). The total volume of trade is reduced; the exact effect will depend on the shape of the two supply and demand curves, the price difference that exists in

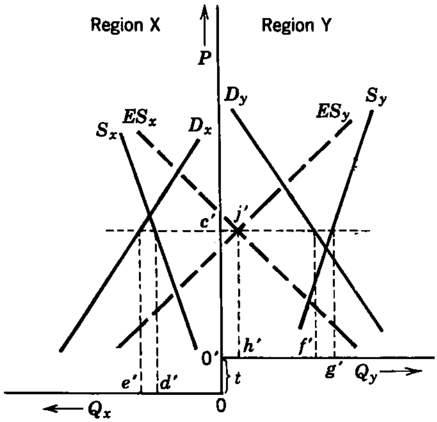


FIGURE 5.4 The effects of transfer cost (t) on prices and trade (compare with Figure 5.2)

the absence of trade, and the magnitude of the transfer cost. However, trade will be possible and profitable as long as the original difference in price is greater than the transfer costs.

A modification of Figure 5.2 will illustrate more clearly the relationship between transfer cost and trade movements. We reproduce in Figure 5.5 the excess supply curves ES_x and ES_y of Figure 5.2. Transfer cost

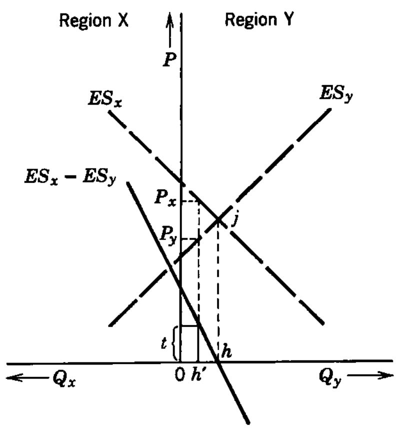


FIGURE 5.5 Equilibrium prices and trade illustrated by using differences between excess supply curves.

can be thought of as a block inserted from the left into the "V" formed by the two excess supply curves, the thickness of the block representing the level of transfer cost. Or, more simply, a new curve can be constructed that represents the vertical difference between the two, $ES_x - ES_y$. The transfer cost can now be measured along the vertical axis, and the quantity traded read off the horizontal axis. In particular, transfer cost of t per unit reduces the quantity shipped from region Y to region X from oh to oh'. A line drawn vertically through the point h' will indicate the equilibrium prices that will exist, namely, P_y and P_x.

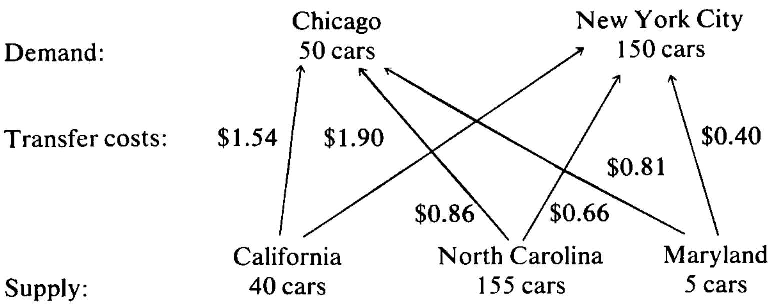
As a corollary to this, it should be apparent that widely separated regions may not become connected by trade because the costs of transportation and handling the product exceed the price differences that exist in the absence of trade. Great distances and expensive transportation thus restrict trade, while technological developments that reduce transfer costs can be expected to increase trade. Before the development of modern highway and rail transportation, for example, trade was concentrated among regions situated along the seacoast or on navigable rivers. Notice also that improvements in transportation increase interregional competition and, in general, reduce the location advantage held by producers situated close to population centers.

The consideration of transportation costs greatly complicates the theoretical analysis of trade. In the absence of these costs, it is a simple matter to aggregate the supply curves and demand curves for many regions, to determine the equilibrium price that will hold in all regions, and to discover which regions will be exporters and which importers. All regions everywhere will be involved, and the actual trade flow patterns connecting exporters and importers will be unimportant, since transfer can be made without cost. With positive transfer costs, however, the multiple-region analysis is more complex. We observed previously that some regions will not be included in the trading system because price differences are less than transfer costs. Moreover, it will now be necessary to determine not only which regions export and which import but also the exact pattern of these flows. In order to combine regional supply curves and demand curves into composites for the single-market trading bloc, we must *know* the precise pattern of trade flows and the related transfer costs, yet, these patterns cannot be known until we complete our analysis! These difficulties will be resolved when we consider more complex location models.

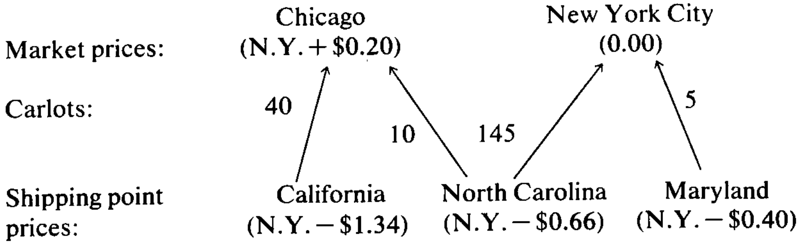
5.3 MULTIREGION MODELS: PRODUCTION AND CONSUMPTION FIXED

A broad class of regional trade problems can be analyzed by using a relatively simple model in which it is assumed that given quantities of a homogeneous product are produced at *m* supply points and that given quantities of the same product are consumed at *n* demand points. Each pair of supply-demand points is connected by transportation facilities over which any amount of the product can be shipped at a given cost per unit which is specified for each pair of trading points.

A simple sketch of a two demand–three supply area problem is presented as a starting point. Let the quantities of cucumbers produced in California, North Carolina, and Maryland equal the unloads at Chicago and New York for a typical week—in this case, 200 carlots. Transfer costs are shown in dollars per bushel from each supply area to each market. As

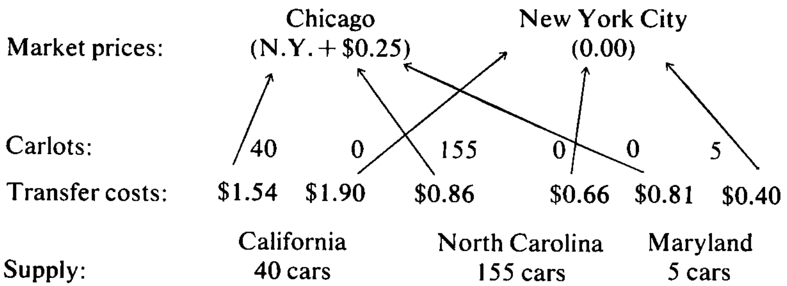


a tentative solution to the problem of selecting the optimum allocation of cucumbers, let California ship 40 cars to Chicago, Maryland ship 5 cars to New York City, and let North Carolina divide its supplies between the two, sending 10 to Chicago and 145 to New York City. Any such allocation implies a specific set of price relationships among producers and markets, as shown below, with New York City chosen as the base.



Markets and production areas are linked by transfer costs over active trading routes. If the trial allocation is, in fact, the optimum, no market will find it profitable to obtain supplies from any other producing area, and no producing area will be able to increase its price by shifting the markets to which it ships. In this example, Chicago is $0.20 above New York. Since Maryland would get $0.20 minus $0.81 or −$0.61 for cucumbers shipped to Chicago compared to −$0.40 in New York, there would be no incentive to shift. The same is true for California in considering shipment to New York, indicating that this allocation is in fact the optimum allocation.

Consider now the situation in which market price differences are given and notice the effect on alternative allocations of supplies. If Chicago is $0.25 above New York, all North Carolina supplies as well as all California supplies will move to Chicago while New York receives only 5 cars from Maryland. Obviously, this is not a "reasonable" allocation although it is "optimum" with the given market price differences. If Chicago fails to maintain a level $0.20 above New York, it will first lose all North Carolina supplies and eventually (N.Y. − $0.37) will lose California supplies as well.



In short, product flows depend on transfer costs and relative market prices; market prices depend on demand and available supplies; product flows and market prices must be developed simultaneously. This process is described briefly in the following section.

5.4 MULTIREGION MODELS: PROGRAMMING SOLUTION

The "transportation problem," as it is often referred to, is a special case of linear programming which has certain features that allow a shortcut solution. Details of this procedure are not reproduced here in any detail, since excellent treatments are readily available.[1] Instead, emphasis is

[1]See reference to Dorfman, Samuelson, and Solow, for example.

placed on the types of information that may be obtained from the transportation problem formulation.

A larger version of the problem outlined in section 5.3 will illustrate the capabilities of this model. Table 5.1 summarizes transfer costs from 8 producing areas to each of 10 markets. In this example, transfer costs include transportation costs plus cartage charges for hauling cucumbers from rail sidings to wholesale market areas in each city. Total quantities available are shown in the last column (S_i) and quantities demanded are shown in the last row (D_j). Figures in parentheses are shipments which are consistent with the minimization of total transfer costs for all shipments. This optimum allocation is one of the useful pieces of information provided by this model.

Table 5.2 is the equilibrium cost matrix. In this table producer price differentials with sign changed, the $-u_i$'s, appear in the last column and market price differentials, the v_j's, appear in the last row.[2] The sum of these values for each route used in the equilibrium solution, identified in Table 5.2 by[a], must equal the unit cost, c_{ij}, for that route. We may select any supply area or market area as a base and may assign a zero price ($0.00) for that entry. The remaining values in the border row and border column are developed in a stepwise fashion by selecting values such that

$$c_{ij} = v_j + (-u_i)$$

In this example, New York was selected as the base and $0.00 entered in the v_j row. We know that the optimum shipping pattern calls for shipments from North Carolina to New York at a transfer cost of $0.66. Thus the entry for North Carolina is

$$\$0.66 = \$0.00 + (-u_i)$$

or $0.66 for $-u_{NC}$. The Philadelphia price can now be computed by using the North Carolina $-u_i$ as: $0.59 = v_j + $0.66 or $-0.07. This process is continued until all border cells are filled. Finally, the body of Table 5.2 is completed by adding the appropriate $-u_i$ and v_j values computed above.

Two types of information are provided by Table 5.2. The intermarket price comparisons can be made directly from the v_j row. For example, the Boston price is $0.14 above the one in New York. Also, comparisons of the returns in the several producing areas are readily made, although it is first necessary to change the sign of each element in the $-u_i$ column. For example, the California price is $1.44 *below* that in New York. A new

[2]By showing values for $-u_i$ instead of u_i, we are able to simplify the discussion somewhat. Since in the optimum solution it is necessary that $c_{ij} \geqslant v_j - v_i$, with $u_i, v_j \leqslant 0$, we may also write $c_{ij} \geqslant v_j + (-u_i)$. The equality holds true for all active routes and for any alternate routes that would not increase total transfer costs.

TABLE 5.1 Cost-Flow Matrix[a]

Exporting Region	Importing Region (Dollars per Bushel)										Surplus S_i (Carlots)
	Boston	New York	Phila-delphia	Pitts-burgh	Chicago	Atlanta	New Orleans	Denver	Los Angeles	San Francisco	
Salisbury, Maryland	0.57	0.40	0.31 (5)	0.49	0.81	0.77	1.06	1.32	1.75	1.86	5
Aurora, Illinois	0.95	0.86	0.81	0.61	0.21 (10)	0.76	0.90	0.93	1.52	1.54	10
Clinton, North Carolina	0.80 (75)	0.66 (150)	0.59 (55)	0.63 (5)	0.86	0.53	0.87	1.32	1.72	1.86	285
Columbia, South Carolina	0.89	0.76	0.70	0.69 (40)	0.82 (30)	0.40	0.77	1.26	1.69	1.83	70
Thomasville, Georgia	1.07	0.96	0.91	0.90	0.90	0.40 (10)	0.52	1.25	1.60	1.80	10
Sanford, Florida	1.11	1.01	0.96	0.96	1.02 (5)	0.59 (5)	0.75	1.38	1.71	1.85	10
San Antonio, Texas	1.48	1.37	1.33	1.19	1.06 (5)	0.96	0.66 (10)	0.90	1.13	1.33	15
Los Angeles, California	1.83	1.90	1.93	1.76	1.54 (5)	1.58	1.41	1.04 (10)	0.18 (50)	0.54 (15)	80
Deficit D_j (Carlots)	75	150	60	45	55	15	10	10	50	15	485

Source. Richard A. King, "Fixed Production–Fixed Consumption Models," *Interregional Competition Research Methods*, edited by Richard A. King, North Carolina Agricultural Policy Institute, Series 10 (Raleigh 1965), Chapter 2, p. 23.

[a]Optimum shipments expressed in carlots are shown in parentheses in the body of the table.

TABLE 5.2 Equilibrium Cost matrix

Exporting Region	Importing Region (Dollars per Bushel)										Producer Price Differentials, $-u_i$
	Boston	New York	Phila-delphia	Pitts-burgh	Chicago	Atlanta	New Orleans	Denver	Los Angeles	San Francisco	
Salisbury, Maryland	0.52	0.38	0.31[a]	0.35	0.48	0.05	0.08	−0.02	−0.88	−0.52	0.38
Aurora, Illinois	0.25	0.11	0.04	0.08	0.21[a]	−0.22	−0.19	−0.29	−1.15	−0.79	0.11
Clinton, North Carolina	0.80[a]	0.66[a]	0.59[a]	0.63[a]	0.76	0.33	0.36	0.26	−0.60	−0.24	0.66
Columbia, South Carolina	0.86	0.72	0.65	0.69[a]	0.82[a]	0.39	0.42	0.32	−0.54	−0.18	0.72
Thomasville, Georgia	0.87	0.73	0.66	0.70	0.83	0.40[a]	0.43	0.33	−0.53	−0.17	0.73
Sanford, Florida	1.06	0.92	0.85	0.89	1.02[a]	0.59[a]	0.62	0.52	−0.34	0.02	0.92
San Antonio, Texas	1.10	0.96	0.89	0.93	1.06[a]	0.63	0.66[a]	0.56	−0.30	0.06	0.96
Los Angeles, California	1.58	1.44	1.37	1.41	1.54[a]	1.11	1.14	1.04[a]	0.18[a]	0.54[a]	1.44
Market price differentials v_j	0.14	0.00	−0.07	−0.03	0.10	−0.33	−0.30	−0.40	−1.26	−0.90	

Source. Richard A. King, "Fixed Production–Fixed Consumption Models," *Interregional Competition Research Methods*, . . . Chapter 2, p. 25.

[a]Equilibrium cost matrix is derived from these values which correspond to transfer costs for active routes. New York is used as the base region.

base region is easily selected by adding to every element in the $-u_i$ column and the v_j row the quantity that will change to zero the border value for the new base region.

The cost of choosing a nonoptimum route is readily determined from Table 5.3. Elements in this table are obtained by subtracting from an element in Table 5.1 the corresponding element in Table 5.2. This provides a direct comparison of the amount that the "market" is willing to pay for products moving over a given route (Table 5.2) with the cost of providing this transfer (Table 5.1). Imputed prices for routes connecting trading pairs will exactly equal transfer cost and will result in zero entries in Table 5.3. In all other cases, transfer costs will be equal to or will exceed the equilibrium market price differentials. When zeros appear in Table 5.3 for routes that are not in use, this indicates that there are one or more alternate shipping plans which could be used without increasing total transfer costs.

The values in Table 5.3 represent costs that would have to be shared between buyer-seller pairs as a condition of making shipments along the indicated routes and so foregoing the trading opportunities indicated by the optimum solution. An alternative interpretation of these values is that they are "opportunity returns" to be shared between buyers and sellers if shipments that may currently prevail along the indicated routes are discontinued and trading diverted to routes for which opportunity costs are zero.

The usefulness of transportation models is increased when combined with side analyses. Location advantages or disadvantages of shippers in the various supply regions do not usually pass intact to raw material suppliers; differentials in processing or manufacturing costs may either offset or enhance the effects of location. Here is an opportunity to utilize information gained from studies of economies of scale, economies of concentration, and regional differentials in costs of factors used for processing or manufacturing. By summing production cost differentials and location advantages, with due regard to sign and by using the same base region throughout, differentials in net returns to raw material suppliers in the various regions may be obtained. The resulting information has implications for the growth or the decline of the industry in various locations.

Similarly, information from transportation models can be used in margins studies. Differences between prices paid by consumers and the ones received by farmers are partially explained by the location of consumers relative to location of their best sources of supplies. The transportation model effectively isolates contributions of transfer costs to farm-to-consumer margins for the various markets. Processing and

TABLE 5.3 Cost of Using Nonoptimum Routes[a]

Exporting Region	Importing Region (Dollars per Bushel)									
	Boston	New York	Phila-delphia	Pitts-burg	Chicago	Atlanta	New Orleans	Denver	Los Angeles	San Francisco
Salisbury, Maryland	0.05	0.02	0.00	0.14	0.33	0.72	0.98	1.34	2.63	2.38
Aurora, Illinois	0.70	0.75	0.77	0.53	0.00	0.98	1.09	1.22	2.67	2.33
Clinton, North Carolina	0.00	0.00	0.00	0.00	0.10	0.20	0.51	1.06	2.32	2.10
Columbia, South Carolina	0.03	0.04	0.05	0.00	0.00	0.01	0.35	0.94	2.23	2.01
Thomasville, Georgia	0.20	0.23	0.25	0.20	0.07	0.00	0.09	0.92	2.13	1.97
Sanford, Florida	0.05	0.09	0.11	0.07	0.00	0.00	0.13	0.86	2.05	1.83
San Antonio, Texas	0.38	0.41	0.44	0.26	0.00	0.33	0.00	0.34	1.43	1.27
Los Angeles, California	0.25	0.46	0.56	0.35	0.00	0.47	0.27	0.00	0.00	0.00

Source. Richard A. King, "Fixed Production—Fixed Consumption Models," *Interregional Competition Research Methods*, . . . Chapter 2, p. 26.

[a]Transfer cost (Table 5.1) less corresponding element in equilibrium cost matrix (Table 5.2).

distribution costs can be added in side analyses to determine minimum margins consistent with competitive equilibria. When compared with margins observed in the actual commodity market, those studies could provide indications of inefficiencies and/or the absence of effective competition in certain sectors of the commodity market.

5.5 PRODUCTION AND CONSUMPTION VARIABLE[3]

Reactive programming makes it possible to obtain solutions to spatial equilibrium problems by maximizing net returns at each shipping point for specified forms of competition. Each supply point is considered as a shipper and, by evaluating the demand function in each of the outlets, a set of gross prices is established. From these gross prices the appropriate transfer costs are deducted to obtain a set of net prices, and supplies are allocated to the outlets that offer the highest net prices. This process is repeated for each shipper in turn, with each making the most profitable allocation possible. When it is not profitable for any shipper to reallocate his supplies among the outlets, the equilibrium solution has been obtained. If supplies at a particular supply point are so large that the equilibrium net price is zero, the surplus quantity remains unallocated. Given the conditions of perfect competition, the equilibrium solution is such that the net revenue to each shipper at the several supply points has been maximized.

Demand Functions with Uniform Slopes—Predetermined Supplies. We first assume uniform slopes for all demand functions to illustrate the reactive programming procedure. A demand function of the form $P = a + bQ$ is needed for each of the consumer centers. Consider the receipts and the corresponding prices shown in Table 5.4*B* as points on market demand functions. Suppose the results of a demand analysis established that the coefficient b, measuring the effect of quantity on price, was −2.0 in each market. With this information, unique demand functions can be established for each of the centers, since the only unknown is the value of a, which can be readily estimated. For consumer center W, $P = \$150$, $Q = 35$, and $b = -2$. Substituting into the demand equation, $P = a + bQ$, we have: $150 = a + (-2)\ (35)$ or $a = 150 + (2)\ (35) = 220$. The demand equation is thus $P_w = 220 - 2Q_w$. Demand equations for other outlets are established in the same manner and are shown in the last line of Table 5.4*B*.

[3]Based on A. D. Seale, Jr., and Thomas E. Tramel, "Reactive Programming Models," *Interregional Competition Research Methods* . . . Chapter 4, pp. 47–58.

By using the transportation cost data specified in Table 5.4*A* and the allocation in Table 5.4*B* as a starting basis, the equilibrium shipments, receipts, prices, and revenues can be determined. The results are shown in Tables 5.4*C* and *D*.

The inspection of the net prices shown in Table 5.4*D* will reveal that the net returns to each of the individual shippers are at equilibrium levels. For each shipping point, the available supplies were allocated to the outlets offering the highest net price; for multiple outlets, the net prices were equated. There is no incentive for any individual shipper to reallocate supplies because any change in the solution would reduce the net returns to the individual shipper making the change. Thus, it is an equilibrium solution.

TABLE 5.4 Reactive Programming: Demand Functions with Uniform Slopes and Predetermined Supplies

A. Transportation Cost[a]

Shipping Point	Consumer Center (Dollars per Carlot)			
	W	X	Y	Z
A	105	230	180	100
B	90	140	100	175
C	200	140	120	110

B. Initial Situation

Shipping Point	Consumer Center (Shipments in Carlots)				Supplies[a]
	W	X	Y	Z	
A	5				5
B	30	10	20		60
C			15	25	40
Receipts	35	10	35	25	105
Prices	150	180	160	150	–
Demand[a]	$P = 220 - 2Q$	$P = 200 - 2Q$	$P = 230 - 2Q$	$P = 200 - 2Q$	–

TABLE 5.4 Continued

C. Equilibrium Solution

Shipping Point	Consumer Center (Shipments in Carlots)				Supplies
	W	X	Y	Z	
A				5	5
B	35		25		60
C		10	10	20	40
Receipts	35	10	35	25	105
Prices	150	180	160	150	–
Demand	$P=220-2Q$	$P=200-2Q$	$P=230-2Q$	$P=200-2Q$	–

D. Equilibrium Prices and Net Revenue

Shipping Point	Consumer Center (Net Prices, Dollars per Carlot)				Net Revenue
	W	X	Y	Z	
A	45.00	–50.00	–20.00	50.00[b]	250.00
B	60.00[b]	40.00	60.00[b]	–25.00	3,600.00
C	–50.00	40.00[b]	40.00[b]	40.00[b]	1,600.00
Prices	150.00	180.00	160.00	150.00	–

Source. A. D. Seale, Jr., and Thomas E. Tramel, "Reactive Programming Models," *Interregional Competition Research Methods* . . Chapter 4, Table 2, p. 51.

[a]Denotes required input data.

[b]Net prices for shipping routes actually used.

Demand Functions with Different Slopes—Predetermined Supplies. In the previous example, it was assumed that we had access to only one estimate of the effect of quantity on price and that this slope coefficient was appropriate for each of the outlets. Now it is assumed that the effect of quantity on price at each market is different. All input data used earlier are also used here except for the effect of quantity on price. The equilibrium solution for this problem, shown in Table 5.5, is quite different from the previous problem. From an inspection of the net prices, it is clear that an equilibrium solution has been obtained because any reallocation would reduce the net returns to any individual shipper making such a change.

TABLE 5.5 Reactive Programming: Demand Functions with Different Slopes and Predetermined Supplies

A. Transportation Cost[a]

Shipping Point	Consumer Center (Cost per Carlot)			
	W	X	Y	Z
A	105	230	180	100
B	90	140	100	175
C	200	140	120	110

B. Initial Situation

Shipping Point	Consumer Center (Shipments in Carlots)				Supplies[a]
	W	X	Y	Z	
A	5				5
B	30	10	20		60
C			15	25	40
Receipts	35	10	35	25	105
Prices	185	180	125	100	–
Demand[a]	$P=220-1Q$	$P=200-2Q$	$P=230-3Q$	$P=200-4Q$	

C. Equilibrium Solution

Shipping Point	Consumer Center (Shipments in Carlots)				Supplies
	W	X	Y	Z	
A	2.2			2.8	5.0
B	60.0				60.0
C		8.6	22.4	9.0	40.0
Receipts	62.2	8.6	22.4	11.8	105.0
Prices	157.80	182.80	162.80	152.80	–
Demand	$P=220-1Q$	$P=220-2Q$	$P=230-3Q$	$P=200-4Q$	–

TABLE 5.5 Continued

D. Equilibrium Prices and Net Revenue

Shipping Point	Consumer Center (Net Prices, Dollars per Carlot)				Net Revenue
	W	X	Y	Z	
A	52.80[b]	−47.20	−17.20	52.80[b]	264.00
B	67.80[b]	42.80	62.80	−22.20	4,068,00
C	−42.20	42.80[b]	42.80[b]	42.80[b]	1,712.00
Prices	157.80	182.80	162.80	152.80	–

Source. A. D. Seale, Jr., and Thomas E. Tramel, "Reactive Programming Models," *Interregional Competition Research Methods*... Chapter 4, Table 3, p. 52.

[a]Denotes required input data.

[b]Net prices for shipping routes actually used.

Demand and Supply Functions with Different Slopes. Up to this point, we have assumed a fixed supply at each of the supply points. In reality, available supplies are a function of the net prices received by producers, and equilibrium levels of these supplies must also be determined. This determination requires the use of supply functions.

In a competitive situation, the equilibrium level of supplies is the level at which net profits or economic rent is zero. Although there are no undistributed supplies in problems of this type, the cost matrix must include all costs incurred between the supply points and the level of demand at the consumer centers in order to compute the equilibrium levels of supplies.

It is assumed that the results of a demand study in each consumer center and cost studies for each of the supply points are available. The demand functions and the production cost functions (provided by these studies) are shown in Tables 5.6*A* and 5.6*B*. The equilibrium levels of supplies, shipments, and receipts, as well as the prices and revenues, can be simultaneously determined by the procedure outlined earlier. They are shown in Tables 5.6*C* and 5.6*D*. A comparison of the net prices and production costs shows that the equilibrium levels of supplies have been determined.

TABLE 5.6 Reactive Programming: Demand and Supply Functions with Different Slopes

A. Transportation Cost[a]

Shipping Point	Consumer Center (Dollars per Carlot)				Production Cost Functions[a]
	W	X	Y	Z	
A	105	230	180	100	$C=35+3S$
B	90	140	100	175	$C=5+1S$
C	200	140	120	110	$C=0+2S$

B. Initial Situation

Shipping Point	Consumer Center (Shipments in Carlots)				Supplies
	W	X	Y	Z	
A	5				5
B	30	10	20		60
C			15	25	40
Receipts	35	10	35	25	105
Prices	185	180	125	100	–
Demand[a]	$P=220-1Q$	$P=200-2Q$	$P=230-3Q$	$P=200-4Q$	–

C. Equilibrium Solution

Shipping Point	Consumer Center (Shipments in Carlots)				Supplies
	W	X	Y	Z	
A				8.05	8.05
B	60.84		3.31		64.15
C		5.43	16.98	2.17	24.58
Receipts	60.84	5.43	20.29	10.22	96.78
Prices	159.15	189.14	169.15	159.14	–
Demand	$P=220-1Q$	$P=200-2Q$	$P=230-3Q$	$P=200-4Q$	–

TABLE 5.6 Continued

D. Equilibrium Prices and Net Revenue

Shipping Point	Consumer Center (Net Prices, Dollars per Carlot)				Production Cost	Economic Rent
	W	X	Y	Z		
A	−4.98	−100.00	−69.99	0[b]	59.14	0
B	0[b]	−20.00	0[a]	−85.00	69.15	0
C	−89.99	0[b]	0[b]	0[b]	49.14	0
Prices	159.15	189.14	169.15	159.14	–	–

Source. A. D. Seale, Jr., and Thomas E. Tramel, "Reactive Programming Models," *Interregional Competition Research Methods*... Chapter 4, Table 4, p. 54.

[a]Denotes required input data.

[b]Net prices for shipping routes actually used.

5.6 LIMITATIONS OF SINGLE-PRODUCT ANALYSIS

We have now demonstrated how it is possible to extend the analysis of spatially separated regions with perfectly inelastic demand and supply curves (outlined in sections 5.3 and 5.4) to deal with cases where either or both quantity demanded and supplied are treated as a function of price. These mathematical procedures also accomplish what we were unable to do graphically in section 5.2 because of the difficulty of knowing which pairs of regions would be engaged in trade and, therefore, by what amount t the price axes for each exporting region should be shifted upward.

However, the student may have become disturbed by the oversimplified analysis of trade in terms of the supply and demand for a single commodity. Clearly, traders will profit from arbitrage as long as prices in any two regions differ by more than transfer costs. It is also apparent that, as a result, producers in an exporting region Y will receive higher prices, while consumers in an importing region X will benefit from lower prices. But what of producers in X and consumers in Y? Does trade result in a general benefit or work only to the advantage of a few in each region? And what of the payment for the goods imported by X? Apparently, the flow of goods from Y to X means a compensating flow of money from X to Y. Yet, money itself is merely a claim on goods, and where are the goods that the producers in region Y will buy with this added income? An understanding of the operation of exchange, either within a region or among regions, requires a more elaborate analytical device than that afforded by supply and demand curves for a single commodity. We shall be concerned with this elaboration in later chapters.

SELECTED READINGS

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