chapter / 20

MULTIPLE-REGION PRODUCTION AND TRADE MODELS[1]

In this chapter the spatial dimension of resource availability, productive activity, and trade are added to the single-region analysis of Chapter 19. Although they fall short of full general equilibrium systems, the models that are presented are both logically consistent and of proven empirical usefulness.

The first two sections treat the case of a single fixed resource—first by using an output orientation, and second by a resource-use orientation. The emphasis is placed on information that concerns equilibrium prices provided by the dual linear programming solution, especially as it bears on site rents. Models with multiple resource restraints are introduced, followed by models that allow trade in intermediate products. A brief summary of an empirical application is presented and reference is made to other studies that may be of interest to readers who wish to carry out similar production and trade analyses.

[1]Much of the material in this chapter is drawn directly from an unpublished report by Gordon A. King, "Analysis of Location of Agricultural Production and Processing" (University of California, Davis, 1965).

20.1 AN OUTPUT-ORIENTED FORMULATION

The following simple model of regional production, consumption, and trade illustrates several important aspects of spatial equilibrium analysis. Assume that there are two markets for two commodities, each of which may be produced in two producing areas. The production technology consists of one activity for each commodity in each region, or four production activities. Inputs consist of one fixed resource – say, land – and a bundle of other inputs that will be expressed in value terms, or "cost of production excluding land rent." Each process has the properties of combining inputs in fixed proportions, of being divisible, and of exhibiting constant returns to scale. It is assumed that all firms in the two regions are quantity adjusters and that they behave according to the rules of pure competition.

The formulation of this problem parallels exactly that of the general equilibrium case discussed in Chapter 19 with the exceptions that (1) the transportation costs are introduced and (2) the cost of production, excluding land rent, replaces the supply equations. The objective function is stated in order to maximize the net value of output, which is the market value less transfer and production costs. The primal problem is illustrated in Table 20.1.

The relationships in Table 20.1 may be restated as follows.

TABLE 20.1 Maximization of Net Value of Output[a]

Rent	Equation Number	Production and Shipment Activity								Restriction R
		Commodity 1				Commodity 2				
		x_{21}^1	x_{31}^1	x_{22}^1	x_{32}^1	x_{21}^2	x_{31}^2	x_{22}^2	x_{32}^2	
	0	N_{21}^1	N_{31}^1	N_{22}^1	N_{32}^1	N_{21}^2	N_{31}^2	N_{22}^2	N_{32}^2	
u_1	1	a_2^1		a_2^1		a_2^2		a_2^2		$\leqslant r_2$
u_2	2		a_3^1		a_3^1		a_3^2		a_3^2	$\leqslant r_3$
u_3	3	-1	-1							$\leqslant -D_1^1$
u_4	4			-1	-1					$\leqslant -D_2^1$
u_5	5					-1	-1			$\leqslant -D_1^2$
u_6	6							-1	-1	$\leqslant -D_2^2$

[a]The net value of output is net returns per unit of output multiplied by output. Model situation consists of single fixed resource, two final commodities, two producing areas, two markets, and one production process for each product in each region.

Maximize net value of output (market value less transfer and production costs)

$$= \sum_i \sum_j \sum_k N_{ij}^k x_{ij}^k = \sum_i \sum_j \sum_k P_{ij}^k x_{ij}^k - \sum_i \sum_j \sum_k t_{ij}^k x_{ij}^k - \sum_i \sum_j \sum_k q_i^k x_{ij}^k$$

where $x_{ij}^k \geqslant 0$.

Subject to

Demand and Supply of Resource by Region

$$(1)\quad a_2^1 x_{21}^1 + a_2^1 x_{22}^1 + a_2^2 x_{21}^2 + a_2^2 x_{22}^2 \leqslant r_2$$
$$(2)\quad a_3^1 x_{31}^1 + a_3^1 x_{32}^1 + a_3^2 x_{31}^2 + a_3^2 x_{32}^2 \leqslant r_3.$$

These equations require that the availability of a resource in a region must equal or exceed production requirements for all commodities.

Demand and Supply of Final Commodity

$$(3)\quad -x_{21}^1 - x_{31}^1 \qquad\qquad \leqslant -D_1^1$$

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. . . .

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$$(6)\quad -x_{22}^2 - x_{32}^2 \qquad\qquad \leqslant -D_2^2.$$

These equations require that the supply of a final commodity produced and shipped to market (1 or 2) in equilibrium must equal the quantity demanded at the specified market price.

The following notation has been used in Table 20.1.

x_{ij}^k = quantity of kth final commodity produced and shipped from i to j.
N_{ij}^k = net return per unit of output of kth final commodity produced in region i and shipped from i to j.
P_j^k = price per unit for commodity k at market location j.
t_{ij}^k = transfer cost per unit of commodity k shipped from i to j.
a_i^k = resource input required per unit of output of k in region i.
q_i^k = cost of production (exclusive of fixed resource) for one unit of the kth commodity in the ith region.

Notice that in the primal problem the objective functions contains, in place of the price of each product, the *net return per unit of output* for the kth commodity produced in region i and shipped from i to j. However, market prices must be consistent with a demand function for the final product, and the associated quantity for each product in a given market is inserted as a restriction. This procedure has been shown by Takayama and Judge (1964) to be consistent with a quadratic programming formulation with the demand function specified in the objective function and has proven to be satisfactory for quantitative work with partial equilibrium models.

We find it useful to consider briefly the dual formulation of this problem, which deals directly with the prices or rents associated with the resource and market restrictions. The dual is sketched in Table 20.2.

TABLE 20.2 Minimization of Returns to Resources[a]

Equation Number	Rents						Restriction R
	u_1	u_2	u_3	u_4	u_5	u_6	
0	r_2	r_3	$-D_1{}^1$	$-D_2{}^1$	$D_1{}^2$	$D_2{}^2$	
1	$a_2{}^1$		-1				$\geqslant N^1_{21}$
2		$a_3{}^1$	-1				$\geqslant N^1_{31}$
3	$a_2{}^1$			-1			$\geqslant N^1_{22}$
4		$a_3{}^1$		-1			$\geqslant N^1_{32}$
5	$a_2{}^2$				-1		$\geqslant N^2_{21}$
6		$a_3{}^2$			-1		$\geqslant N^2_{31}$
7	$a_2{}^2$					-1	$\geqslant N^2_{22}$
8		$a_3{}^2$				-1	$\geqslant N^2_{32}$

[a]This is the dual problem to the one in Table 20.1.

We may write out the relationships shown in Table 20.2 as follows. Minimize returns to resources

$$= u_1 r_1 + u_2 r_2 + u_3 D_1{}^1 + u_4 D_2{}^1 + u_5 D_1{}^2 + u_6 D_2{}^2$$

where $u_i \geqslant 0$.

Subject to

$$\begin{aligned}
&(7)\quad u_1 a_2{}^1 - u_3 \geqslant N^1_{21} = P_1{}^1 - t^1_{21} - q_2{}^1\\
&(8)\quad u_2 a_3{}^1 - u_3 \geqslant N^1_{31} = P_1{}^1 - t^1_{31} - q_3{}^1\\
&(9)\quad u_1 a_2{}^1 - u_4 \geqslant N^1_{22} = P_2{}^1 - t^1_{22} - q_2{}^1\\
&(10)\quad u_2 a_3{}^1 - u_4 \geqslant N^1_{32} = P_2{}^1 - t^1_{32} - q_3{}^1\\
&(11)\quad u_1 a_2{}^2 - u_5 \geqslant N^2_{21} = P_1{}^2 - t^2_{21} - q_2{}^2\\
&(12)\quad u_2 a_3{}^2 - u_5 \geqslant N^2_{31} = P_1{}^2 - t^2_{31} - q_3{}^2\\
&(13)\quad u_1 a_2{}^2 - u_6 \geqslant N^2_{22} = P_2{}^2 - t^2_{22} - q_2{}^2\\
&(14)\quad u_2 a_3{}^2 - u_6 \geqslant N^2_{32} = P_2{}^2 - t^2_{32} - q_3{}^2
\end{aligned}$$

Equations 7 through 14 state that net returns at each producer location should be equal to unit rent for the corresponding resource. Net returns per unit of output at a producer location are equal to rent to the fixed resource for one activity in the program. The value u_1 is rent per unit fixed resource, which gives rent per unit of output when multiplied by the input-output coefficient. Each of the values u_3 through u_6 is an artificial rent that will equal zero when assumed market price is equal to equilibrium market price.

Equations that are associated with the dual problem, as given in Table 20.2, provide a clearer statement as to the relationship between net returns and rents, u_1 and u_2, that accrue to the fixed resource by region. There are four possible activities associated with land rent in each region, as shown in the first two columns of the dual (or the first two equations in the primal). Take region 1 as an example. Commodity 1 can be produced and then shipped either to market 1 or market 2. A similar procedure is followed for commodity 2. In the solution, the rent associated with the fixed resource, land, will be determined by the activity or activities selected in the program. In equilibrium, the rent associated with land in region 1 will be equal for any of the activities that are included in the solution: that is, for the u_1 in Equations 1, 3, 5, or 7 given in Table 20.2. As pointed out, the imputed values of u_3 through u_6 will be zero with the demand price-quantity equilibrium conditions. The solution that maximizes the net value of output also minimizes returns to the fixed resource, which is the well-known outcome of linear programming problems.

In matrix notation the direct and dual problems of Tables 20.1 and 20.2 may be written as follows.

Direct

Maximize Px

Subject to

$$x \geqslant 0$$
$$Ax \leqslant R$$

Dual

Minimize uR

Subject to

$$u \geqslant 0$$
$$uA \geqslant P$$

where P is a vector of prices at region of production; x is a vector of choice variables for production and shipment; R is a vector of restraints; A is a matrix of input-output coefficients; and u is a vector of imputed prices or rents to fixed factors.

20.2 A RESOURCE-ORIENTED FORMULATION

With the single resource restraint, an alternative formulation of the objective of maximization of net value of output may be of interest. This can be expressed as net return per unit fixed resource multiplied by the quantity of the fixed resource. This quantity is equivalent to the net returns per unit of output multiplied by total output when constrained by resource availability and quantity demanded. The formulation of the problem in this way is given in Table 20.3 and approximates the one suggested by Beckmann and Marschak (1955) as a representation of a discrete production von Thünen model. The researcher may encounter a variety of problems where it is appropriate to simplify to a single resource, single production process case.

TABLE 20.3 Resource-Oriented Approach to Maximization of Net Value of Output[a]

Rents	Eq. No.	Production and Shipment Activity								Restriction
		Commodity 1				Commodity 2				
		r^1_{21}	r^1_{31}	r^1_{22}	r^1_{32}	r^2_{21}	r^2_{31}	r^2_{22}	r^2_{32}	
	0	n^1_{21}	n^1_{31}	n^1_{22}	n^1_{32}	n^2_{21}	n^2_{31}	n^2_{22}	n^2_{32}	
u_1	1	1				1				$\leqslant r_2$
u_2	2		1				1			$\leqslant r_3$
u_3	3	$-y_2{}^1$	$-y_3{}^1$							$\leqslant -D_1{}^1$
u_4	4			$-y_2{}^1$	$-y_3{}^1$					$\leqslant -D_2{}^1$
u_5	5					$-y_2{}^2$	$-y_3{}^2$			$\leqslant -D_1{}^2$
u_6	6							$-y_2{}^2$	$-y_3{}^2$	$\leqslant -D_2{}^2$

[a]Net returns per unit fixed resource multiplied by fixed resource.
Note. Assumptions are single fixed resource, two products, two producing areas, two markets, and one production process for each product in each region.

The relationships described in Table 20.3 can be summarized as follows.

We wish to maximize the net value of output (net return per unit fixed resource multiplied by a quantity of fixed resource) which is

$$\sum_i \sum_j \sum_k n^k_{ij} r^k_{ij} = \sum_i \sum_j \sum_k \left[(P_j{}^k - t^k_{ij} - q_j{}^k) y^k \cdot r^k_{ij} \right]$$

where $r^k_{ij} \geqslant 0$.

Subject to

Demand and Supply of Resource by Region

$$(1)\quad 1 \cdot r^1_{21} + 1 \cdot r^2_{21} \leqslant r_2$$
$$(2)\quad 1 \cdot r^1_{31} + 1 \cdot r^2_{31} \leqslant r_3$$

The availability of fixed resource must equal or exceed requirements for producing commodities 1 and 2 in region 2 (F_2) or in region 3 (F_3).

Demand and Supply of Final Commodity

$$(3) -y_2{}^1 r^1_{21} - y_3{}^1 r^1_{31} \leqslant -D_1{}^1$$

. . . .

$$(6) -y_2{}^2 r^2_{22} - y_3{}^2 r^2_{32} \leqslant -D_2{}^2$$

The supply of final commodity produced and shipped to market (1 or (2) in equilibrium must equal the quantity demanded at the specified market price $P_j{}^k$.

The following notation has been used.

r^k_{ij} = quantity of fixed resource used to produce the kth final commodity in region i and shipped to region j.

n^k_{ij} = net return *per unit of fixed resource* used to produce the kth final commodity in region i and shipped from region i to j (equal to net return per unit output divided by input-output coefficient).

$y_i{}^k$ = output per unit of input for production of kth commodity in region (equal to reciprocal of the input-output coefficient $a_i{}^k$ in Table 20.1).

20.3 LOCATION RENTS

Our discussion thus far has considered land in a given region to be homogeneous, and the rents derived are strictly location rents. A move toward reality would be to introduce land of varying quality in the restraints. The imputed rents would then be in terms relevant to the particular quality of land in the given region. Actually, land quality varies even on a given field on one producing unit, and the gain in reality would be open to debate. The point to be reemphasized here is that a given rent structure—for instance, around a given market—depends on the market price, the transfer costs for the product, the quality differentials in land, and the nonland costs of production. It will be a smooth function only with homogeneous land, constant factor prices, and transfer costs that are continuous with distance.

The introduction of the variable, nonland costs of production, replaces several relations that would be included in a general equilibrium framework discussed in the previous chapter. What has been done is to lump all nonland costs into a single figure instead of specifying in a production relation the inputs of resources other than land and any intermediate commodities used in production and then, by assuming prices of these items are given and fixed, to obtain the value of nonland costs.

Let us designate the *nonland cost per acre* as (c^k), which is equal to yield per acre (y^k) multiplied by the nonland cost of production for one unit of the kth commodity (q^k). The supply function for this bundle of inputs is thus perfectly elastic at the determined cost, as shown in Figure 20.1. The supply function for land is perfectly inelastic at the given level (r). For this single commodity case, land rent is generated only if the available supply is employed; and the level of rent is a function of the demand for the product.

In the short run, the supply function for land is relatively inelastic. But in the long run, new areas may be brought into production (within limits) through irrigation projects, reclamation of alkaline soil, clearing, and so forth. Let us assume for the moment that in the area under consideration all land is under cultivation, so that an inelastic supply function for the region is relevant.

Suppose that we wish to determine how the present pattern differs from that of an efficient spatial production pattern. Another way of stating the problem is to ask: What is the level of land rent for a particular location as compared with that under the "optimum" pattern? We consider first the determination of the equilibrium land rent and then discuss some of the problems of such a comparison with "present" land rents.

Let production technology for each individual crop or crop combination be represented by three alternative technologies that consist of different combinations of land and nonland inputs per unit of output (Figure 20.2). Notice that yield per acre is the reciprocal of the coefficient (a^k), the land input per unit of output. Therefore, yield and nonland costs are positively related in a linear or nonlinear fashion. The exact form of this relationship is vital in determining the production process selected for each crop and region.

In this formulation, land costs are taken to be zero, and we obtain imputed rent to the single fixed resource, land. As was done previously, with given transfer costs and an assumed price in each market, we proceed to maximize net value of output subject to resource and quantity demanded restraints. The dual provides the rent per acre for a particular

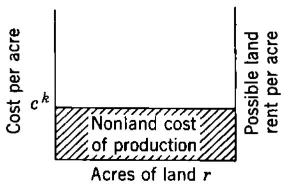

FIGURE 20.1 The land rent per acre under a simplified case.

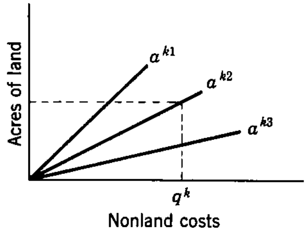

FIGURE 20.2 The production activities for one unit of output.

commodity k and process s, one of which will be a maximum for a particular piece of land. This can be expressed as

$$u^{ks} \qquad a^{ks} \quad \geq \quad N^k \quad = \quad P^k \quad - \quad t^k_{21} \quad - \quad q^{ks}$$

$$\begin{bmatrix}\text{Rent}\\ \text{per}\\ \text{acre}\end{bmatrix}\begin{bmatrix}\text{Land per}\\ \text{unit of}\\ \text{output}\end{bmatrix} \geq \begin{bmatrix}\text{Net return}\\ \text{per unit}\\ \text{of output}\end{bmatrix} = \begin{bmatrix}\text{Market}\\ \text{price}\\ \text{per unit}\\ \text{of output}\end{bmatrix} - \begin{bmatrix}\text{Transfer}\\ \text{cost per}\\ \text{unit of}\\ \text{output}\end{bmatrix} - \begin{bmatrix}\text{Nonland}\\ \text{cost per}\\ \text{unit of}\\ \text{output}\end{bmatrix}$$

With a given price at a producer's location (that is, $P_j^k - t_{ij}^k$), there will be only one relevant production process for each commodity that will provide maximum returns. However, if we do not know to which market commodities will be shipped, net producer prices will not be known. Similarly, the levels of the market prices will not be known. Thus, we cannot specify in advance which production activity should be included in the program. It would be expected that the areas located farther from the market (and thus having lower net price per unit of output because of higher transfer costs) would select production processes requiring relatively more land and less nonland costs, with the associated lower yield per acre. In any case, our equilibrium model will provide the imputed land rent per acre (u^{ks}) associated with the kth commodity and the production process or system s.

These rents would correspond to actual market rents under very restrictive conditions of a static framework in which the land rental market was strictly competitive. Actual land rents and associated production patterns could be compared with the ideal production patterns and imputed rents. If land were rented out to farmers rather than following the typical ownership pattern, producers would treat land rents as a cost per acre; and, in a perfectly competitive situation, these rentals would correspond to the imputed rents. Also, the production pattern, selection of activities, and shipment patterns would correspond to the equilibrium solution.

In looking at data from actual production patterns, the rental levels and the production activities reflect adjustments to actual market prices, transfer costs, mill locations for processed commodities, and shipment patterns. Selection of given levels of nonland costs for use in an equilibrium model raises theoretical questions, especially when these nonland costs might include intermediate commodities that are not produced in the region but are shipped in. Thus the level of these costs will vary depending on the distance shipped, for example, which may depend on the absolute level of production in a particular region. Thus, the assumed supply function that is completely elastic at a given cost level may be relevant for the analysis of certain types of agricultural production but would not be general enough for others.

20.4 MULTIPLE RESOURCE RESTRAINTS

This production model is easily modified to include multiple resource restraints as well as alternative production processes. The objective function is now formulated so as to maximize the net value of output defined as net returns per unit of output multiplied by output. Table 20.4 is modified to include two production processes and two resource restraints for each region. For simplicity, production processes for commodity 1 only are shown here. Notice that the input-output coefficients

TABLE 20.4 Alternative Production Processes With Multiple Resource Restraints

Equation Number	Production and Shipment Activities for Single Commodity in Two Regions								Restraints
	Production in Region 2				Production in Region 3				
	x_{21}^{11}	x_{21}^{12}	x_{22}^{11}	x_{22}^{12}	x_{31}^{11}	x_{31}^{12}	x_{32}^{11}	x_{32}^{12}	
0	N_{21}^{11}	N_{21}^{12}	N_{22}^{11}	N_{22}^{12}	N_{31}^{11}	N_{31}^{12}	N_{32}^{11}	N_{32}^{12}	
1	a_2^{111}	a_2^{112}	a_2^{111}	a_2^{112}					$\leqslant r_2^1$
2	a_2^{211}	a_2^{212}	a_2^{211}	a_2^{212}					$\leqslant r_2^2$
3					a_3^{111}	a_3^{112}	a_3^{111}	a_3^{112}	$\leqslant r_3^1$
4					a_3^{211}	a_3^{212}	a_3^{211}	a_3^{212}	$\leqslant r_3^2$
5	−1	−1			−1	−1			$\leqslant -D_1^1$
6			−1	−1			−1	−1	$\leqslant -D_1^2$

a_i^{fks} have superscripts indicating the amount of resource f required per unit of output of the kth commodity produced by process s. The subscript refers to region of production.

20.5 TRADE IN INTERMEDIATE COMMODITIES

Following Beckmann and Marshak (1955), commodities now are divided in three classes: (1) resources or primary commodities, (2) intermediate commodities, which are produced commodities emerging from a production process as an output and then entering some other process as an input, and (3) commodities that are produced or final commodities desirable in themselves. Resource flows per unit of time include items such as land, labor, and a catchall "capital." Certain resources are fixed in both the long and short runs, such as land, but labor is mobile to some extent in the long run, and migratory farm labor by definition is mobile among regions in the short run. Capital mobility is an interesting topic for research in itself, and we bypass this admittedly difficult area, only suggesting its importance for "further work" in models that incorporate technological change and in growth models.

Intermediate commodities and final commodities can be shipped among regions. Notice that the transportation industry "produces" an intermediate commodity of space transfer, and Lefeber (1958) incorporates the transportation production process directly into the spatial equilibrium framework. The scope of intermediate commodity production is very wide. For example, in the Leontief *closed* input–output model, all commodities are considered as intermediate, with final commodities considered as inputs for the production of personal services. The *open* input–output model specifies "final commodity" demand as a separate sector determined outside the system.

In agricultural economics research, there has been considerable specialization of analysis that centers on the production processes for intermediate commodities at the farm level or production processes at the processing level—to cite two areas that have received much attention in the past. Admittedly, it is often useful to make concentrated efforts on one phase and live with "assumed" prices or costs in other sectors. Perhaps the ability to analyze large quantities of data with current computer techniques will encourage more comprehensive approaches to problems that require recognition of the interrelationships among the various sectors of the agricultural complex.

In fact, intermediate commodities can be further divided into various subgroups. Takayama and Judge (1964) refer to *primary* intermediate commodities as the ones "emerging from a farm-level production process as an output and entering some other (production) process as an input—for example, feed grains or feeder cattle." *Secondary* intermediate commodities are defined as the ones that are "produced commodities emerging from a farm-level production process as output and entering the processing activities as an input—for example, commodities such as cattle or hogs which are ready for consumption after they are processed." Rather than specify various levels of intermediate commodities, it seems more appropriate to treat them as a group, except when we may wish to designate a processing production process that requires certain intermediate commodities as well as resources as inputs. Actually, it is hard to imagine any commercial agricultural production that does not require purchased intermediate commodities and, hence, there is no need to press the point further. What we really want to consider here is the implications of a model in which *intermediate commodities may be produced and shipped between regions* and, therefore, it must be considered explicitly in the location of the production process in the next stage toward production of the final commodity.

The shipment of intermediate commodities is particularly important in the feed-livestock economy. This implies that the equilibrium pattern of grain prices cannot be determined without the consideration of the location of livestock production and, similarly, for livestock production. The processing locations for grain and for livestock also must be considered, but it will be deferred to a subsequent section. We thus specify the spatial equilibrium counterpart to the general equilibrium discussion in Chapter 19. For practical reasons, the discussion will center on a model with a single market, two producing areas, each with a single resource restraint for the production of a single intermediate commodity, and two products. This can easily be generalized but at some loss of clarity.

This problem is stated below as a set of equations and in a programming format that parallels previous developments in Table 20.5 and, hence, needs no particular comment. However, it should be noted that the objective function requires selection of not only product prices (P) but also intermediate commodity prices (p). If we added resource supply as a function of price, the situation would become even more difficult. With a single production process, a single market, and only two commodities, selection of prices (P, p) that would be feasible does not appear difficult, utilizing the known interrelationships of unit cost and prices inherent in the dual. But, as the problem grows in size and complexity, the choices would be formidable, based on experience with a one commodity-model that ignored intermediate commodity prices as such.

TABLE 20.5 Spatial Equilibrium Model with Trade in Intermediate Commodities

Equation Number	Final Commodities: Region 2		Final Commodities: Region 3		Intermediate Commodities: Region 2		Intermediate Commodities: Region 3		Restraints
	x^1_{21}	x^2_{21}	x^1_{31}	x^2_{31}	y_{22}	y_{23}	y_{33}	y_{32}	
0	P^1_{21}	P^2_{21}	P^1_{31}	P^2_{31}	p_{22}	p_{23}	p_{33}	p_{32}	
1	a^1	a^2			a^y	a^y			$\leqslant r_2$
2			a^1	a^2			a^y	a^y	$\leqslant r_3$
3	b^1	b^2			1	−1		1	$\leqslant s_2$
4			b^1	b^2		1	1	−1	$\leqslant s_3$
5	−1		−1						$\leqslant -D^1$
6		−1		−1					$\leqslant -D^2$

Note. This model includes two final commodities, one market, one intermediate commodity, one resource, and two producing regions.

The following notation has been used in Table 20.5.

x^k_{ij} = quantity of kth final commodity produced and shipped from i to j.
y_{ij} = quantity of intermediate commodity produced and shipped from i to j.
P^k_{ij} = price per unit of kth commodity at production region i if shipped to region j, and equal to market price P_j less transfer cost t_{ij}.
p_{ij} = price per unit of intermediate commodity at region j; if $i = j$ and $t_{ii} = 0$, then net and gross price are equal, with $i \neq j$ equal to $p_i - t_{ij}$.
a^i = resource input per unit output of final commodity.
b^i = intermediate commodity input per unit output of final commodity.
r_i = quantity of resource available in ith region.
s_i = quantity of intermediate commodity *produced* in region i.
D^k = quantity of commodity k consumed at price P_j^k

The objective here is to maximize the net value of output, subject to certain restraints. The usual competitive conditions are assumed, with transportation cost between regions given and independent of volume shipped. This is an assumption that requires further attention in itself. We divide the restraints into three groups: the first are the ones used in the direct; the second, the ones used in the dual; and the third, the implicit functions related to the prices introduced into the objective function.

For the primal problem, we may write the following equations (see also Table 20.5).

Resource Demand and Supply (assumed given by region)

$$(1)\quad a^1x_{21}^1 + a^2x_{21}^2 + a^yy_{22} + a^yy_{23} \leq \bar{r}_2$$
$$(2)\quad a^1x_{31}^1 + a^2x_{31}^2 + a^yy_{33} + a^yy_{32} \leq \bar{r}_3$$

Intermediate Commodity Demand and Supply

$$(3)\quad b^1x_{21}^1 + b^2x_{21}^2 + 1 \cdot y_{22} - 1 \cdot y_{23} + 1 \cdot y_{32} \leq S_2 = s_2 \pm \text{shipments}$$
$$(4)\quad b^1x_{31}^1 + b^2x_{31}^2 + 1 \cdot y_{23} + 1 \cdot y_{33} - 1 \cdot y_{32} \leq S_3 = s_3 \pm \text{shipments}$$

Final Commodity Supply and Demand

$$(5)\quad -1 \cdot x_{21}^1 - 1 \cdot x_{31}^1 \leq -D^1$$
$$(6)\quad -1 \cdot x_{21}^2 - 1 \cdot x_{31}^2 \leq -D^2$$

The dual conditions of equilibrium can be written as follows.

Final Commodity Price Equal to Unit Cost (Intermediate commodity plus imputed cost of resource for activity included.)

Intermediate Commodity Price Equal to Unit Cost (Imputed cost of resource for activity included. Obviously, several resources and other "intermediate commodities" are required under a more realistic situation.)

The following side analyses are used in establishing price-quantity relationships.

Demand for Final Commodity

$$(1)\quad D^1 = f(P_1^1 P_1^2 \ldots)$$
$$(2)\quad D^2 = f(P_1^1 P_1^2 \ldots)$$

Supply of Intermediate Commodity

$$(1)\quad s_2 = F(P_1^1 - t_{21}, P_1^2 - t_{21}, \ldots)$$
$$(2)\quad s_3 = F(P_1^1 - t_{31}, P_1^2 - t_{31}, \ldots)$$

Clearly, a problem of a size having relevance for decision-makers would be very large and complex. Our purpose here has been to illustrate how various research problems have been formulated to indicate some of the implicit assumptions of partial equilibrium analyses. Realistically, quantification of spatial models will be less than general, and equilibrium will never be fully specified. But the insights gained by these partial analyses hold considerable promise. Since the types of simplifying assumptions that are made for one kind of production analysis may not be relevant for another, we have not singled out any particular model as most suitable. That is the first task of the economist who ventures into the unknown of applied economics.

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