ACE 564
Spring 2006

Lecture 3

The Multiple Regression Model: Specification and Estimation

by
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Readings:

Griffiths, Hill and Judge. "Model Specification and the Data," Section 9.1 and "Estimation," Section 9.2 in *Learning and Practicing Econometrics*

Introduction to the Multiple Regression Model

The simple linear regression model is useful in numerous situations

In practice, most problems involve two or more explanatory variables

A regression model with more than one explanatory variable is called a multiple regression model

In its most general form, the multiple regression model can be stated as,

$$y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + \ldots + \beta_k x_{k,t} + e_t$$

where:

- $k = 1, \ldots, K$ and $t = 1, \ldots, T$
- The $K\text{-}1$ explanatory variables are $x_{t,2}$ through $x_{t,k}$
- β_1 measures the value of the dependent variable when all of the explanatory variables equal zero

- Interpretation of the *K-1* slope parameters, β_2 through β_k, requires the development of some new concepts

To develop the needed concepts, it is helpful to return to a discussion of the nature of scientific experiments

- An experiment is the use of data to test the validity of a hypothesis drawn from some underlying theory

- In this general sense, all applied research consists of experiments

- There are important distinctions regarding how experimental data are generated

Example of Experimental Research

A plant scientist is interested in measuring the effect of herbicide ("weed killer") use on corn yield

The research objective can be expressed in implicit form as,

$$y = f(H)$$

where y is corn yield and H is herbicide application

Underlying theory identifies several variables, in addition to herbicide, that affect corn yield,

$$y = f(H, F, W, C)$$

where

y = corn yield
H = rate of herbicide application
F = level and composition of fertilizer
W = water availability during growing season
C = cultural practices during planting, growing, harvesting

Theory makes it clear that researcher must control for the effect of all four variables in order to correctly identify separate effect of herbicide application

The research objective can now be correctly stated as,

$$y = f(H \mid F, W, C)$$

which is interpreted as measuring the effect of herbicide on yield, "controlling" for the effect of F, W, and C on corn yield

No. 1 $H = 5$	No. 2 $H = 15$	No. 3 $H = 5$
No. 4 $H = 10$	No. 5 $H = 15$	No. 6 $H = 10$
No. 7 $H = 15$	No. 8 $H = 5$	No. 9 $H = 10$

Underlying theory: $Y = f(H, F, W, C)$

Research equation: $Y = f(H \mid F, W, C)$

where

- Y = corn yield
- H = herbicide application
- F = fertilizer level
- W = water availability
- C = cultural practices on the plots

Research objective: Measure the yield–herbicide relationship.

Figure 4.1 Illustration of an Experimental Design to Measure the Effect of Herbicide on Corn

Johnson, Aaron C. Jr., Marvin B. Johnson, Rueben C. Buse. Econometrics: Basic and Applied. MacMillan Publishing Company. New York.1989.

Table 4.1. Worksheet Used to Record Yield Data

Level of Herbicide, H	*Yield of Subplots*	*Average Yield by Herbicide Level*
5	Y_1, Y_3, Y_8	$\overline{Y}_5$
10	Y_4, Y_6, Y_9	$\overline{Y}_{10}$
15	Y_2, Y_5, Y_7	$\overline{Y}_{15}$

Johnson, Aaron C. Jr., Marvin B. Johnson, Rueben C. Buse. Econometrics: Basic and Applied. MacMillan Publishing Company. New York.1989.

The corn yield example demonstrates the "typical" understanding of experimental research

Researcher generates the data under experimental conditions that control for (or hold constant) the effect of all variables except the one of interest

Social science research cannot be viewed as an experimental science in this sense, because researcher seldom (if ever) generates data under controlled conditions

Instead, social science research must use observational data generated by the system being studied

- Researcher, therefore, has no control over data generation

- "After the fact," must somehow control for effect of all "important" variables in order to identify relationship under study

Examples of non-experimental sciences:

Economics, Astronomy, Geology, Evolutionary Biology

Example of Non-Experimental Research

An agricultural economist is interested in measuring effect of income change on the quantity of food consumption,

$$Q = f(I)$$

Based on the theory of consumer choice, formulate the following "complete" model,

$$Q = f(I, PF, PNF)$$

where:

Q	= quantity of food consumption
I	= consumer income
PF	= price of food
PNF	= price of non-food goods

The specific research question is to determine,

$$Q = f(I \mid PF, PNF)$$

or, in words, measure the effect of income on food consumption, holding constant ("controlling") the effect of *PF* and *PNF*

It is not possible to randomly assign consumers to subgroups that vary by income and hold constant the price of food, and the price of non-food

$\Rightarrow$ Experimental control in the usual sense is not possible

Instead, we assume that we can write down the "correct" statistical (econometric) model that generates observable data on the quantity of food, given income, the price of food and the price of non-food

- Assumes that the variables and functional form are specified correctly

- Another place where the issue of specification error arises

Let's say the researcher is willing to assume a linear functional form

Then the "true" model that generates observable data is,

$$Q = \beta_1 + \beta_2 I + \beta_3 PF + \beta_4 PNF$$

Note that effect of all independent variables on Q is determined simultaneously

However, the researcher also knows that it is unlikely that actual observations will fit this model exactly

These "random" influences are aggregated into an error term that varies each time period,

$$Q_t = \beta_1 + \beta_2 I_t + \beta_3 PF_t + \beta_4 PNF_t + e_t$$

The above model is assumed to generate the observed sample values of the quantity of food demanded

Now, let's interpret the meaning of the slope parameters

Take the expectation of the model,

$$E(Q_t) = \beta_1 + \beta_2 I_t + \beta_3 PF_t + \beta_4 PNF_t$$

Next, fix values for *PF* and *PNF* to obtain the conditional expectation,

$$E(Q_t \mid \overline{PF}_t, \overline{PNF}) = \beta_1 + \beta_2 I_t + \beta_3 \overline{PF}_t + \beta_4 \overline{PNF}_t$$

By defining $\beta_1' = \beta_1 + \beta_3 \overline{PF}_t + \beta_4 \overline{PNF}_t$ we can re-write this relationship as,

$$E(Q_t \mid \overline{PF}_t, \overline{PNF}_t) = \beta_1' + \beta_2 I_t$$

which measures the conditional mean of food consumption (Q_t) as a function of income (I_t), holding constant the effect of *PF* and *PNF*

The change in the conditional mean of Q_t for a one-unit change in I_t is,

$$\frac{\Delta E(Q_t \mid \overline{PF}_t, \overline{PNF}_t)}{\Delta I_t} = \beta_2$$

This relationship holds for any fixed values of *PF* and *PNF*

The definition of β_2 is now clear:

> the change in "average" food consumption for a one-unit change in income, holding constant the level of *PF* and *PNF*

We can find the same result with the use of calculus,

$$\frac{\partial E(Q_t)}{\partial I_t} = \beta_2$$

Note that we do not have to indicate directly that *PF* and *PNF* are held constant, this is implicit in the use of the partial differential notation

Likewise, $$\frac{\partial E(Q_t)}{\partial PF_t} = \beta_3 \qquad \frac{\partial E(Q_t)}{\partial PNF_t} = \beta_4$$

General Interpretation

This non-experimental example provides a basis for general interpretation of parameters on independent variables ("slope" parameters) in multiple regression models

- The coefficient β_k for $x_{k,t}$ measures the change in the conditional mean of y_t for a one-unit change in $x_{k,t}$, holding constant the effect on y_t of the other independent variables in the regression

- Sometimes β_k is referred to as a partial regression coefficient

Now, we can tie together the non-experimental and experimental approaches

Assuming the regression model is correct, the estimated slope parameter of the multiple regression will provide the same measure of the conditional effect of an explanatory variable on the dependent variable as if the researcher had conducted a randomized experimental test

Finally, in the multiple regression model the intercept parameter, β_1, is the value of the dependent variable when each of the independent, explanatory variables takes the value zero

In many cases this parameter has no clear economic interpretation, but it is almost always included in the regression model

Generally, it helps in the overall estimation of the model and in forecasting

For a further (and entertaining) discussion of the differences and similarities between experimental and non-experimental research, I highly recommend:

> Leamer, E.E. "Let's Take the Con out of Econometrics." *American Economic Review*, 73(1983):31-43.

Specification of the Multiple Regression Model

The Problem

Bay Area Rapid Food is a hamburger chain

Each week management must decide:

- How much money to spend on advertising
- Whether to lower prices as special promotions

Key Questions

- How does total revenue change as the level of advertising expenditure changes?
- Does an increase in advertising expenditure lead to an increase in total revenue?
- If so, is the increase in total revenue sufficient to justify the increased advertising expenditure?
- Will reducing prices lead to an increase or decrease in total revenue?

- If a reduction in price leads a decrease in total revenue then demand is price inelastic; If a price reduction leads to an increase in total revenue then demand is price elastic

Hence, management needs to know the relationship between total chain revenue, advertising and prices

The Economic Model

Define,

y: total revenue each week
x_2: price of food and beverage products
(an average that describes overall prices)
x_3: advertising expenditures for the week

Economic theory suggests that,

$$y = f(x_2, x_3)$$
$$\pm \; +$$

We need to be more precise about the relationship in order to estimate the parameters of the relationship

For simplicity, let's assume a linear relationship is reasonable,

$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3$$

We could have assumed one of the other functional forms considered in Lecture 1

The Statistical Model

The linear economic model predicts that total revenue for a given level of price and advertising will be the same for all weeks

Recognize that actual total revenue for a given level of price and advertising will not be the same for all weeks,

$$y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + e_t \qquad t = 1,\ldots,T$$

Note that $E(y_t) = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t}$ forms a plane and the data points scatter in three dimensions around this plane

Table 7.1 **Weekly Observations on Revenue, Price, and Advertising Expenditure for the Hamburger Chain**

Week	Revenue (*tr*) $1,000 units	Price (*p*) $	Advertising (*a*) $1,000 units
1	123.1	1.92	12.4
2	124.3	2.15	9.9
3	89.3	1.67	2.4
4	141.3	1.68	13.8
5	112.8	1.75	3.5
6	108.1	1.55	1.8
7	143.9	1.54	17.8
8	124.2	2.10	9.8
9	110.1	2.44	8.3
10	111.7	2.47	9.8
11	123.8	1.86	12.6
12	123.5	1.93	11.5
13	110.2	2.47	7.4
14	100.9	2.11	6.1
15	123.3	2.10	9.5
16	115.7	1.73	8.8
17	116.6	1.86	4.9
18	153.5	2.19	18.8
19	149.2	1.90	18.9
20	89.0	1.67	2.3
21	132.6	2.43	14.1
22	97.5	2.13	2.9
23	106.1	2.33	5.9
24	115.3	1.75	7.6
25	98.5	2.05	5.3
26	135.1	2.35	16.8
27	124.2	2.12	8.8
28	98.4	2.13	3.2
29	114.8	1.89	5.4
30	142.5	1.50	17.3
31	122.6	1.93	11.2
32	127.7	2.27	11.2
33	113.0	1.66	7.9
34	144.2	1.73	17.0
35	109.2	1.59	3.3
36	106.8	2.29	7.1
37	145.0	1.86	15.3
38	124.0	1.91	12.7
39	106.7	2.34	6.1
40	153.2	2.13	19.6
41	120.1	2.05	6.3
42	119.3	1.89	9.0
43	150.6	2.12	18.7
44	92.2	1.87	2.2
45	130.5	2.09	16.0
46	112.5	1.76	4.5
47	111.8	1.77	4.3
48	120.1	1.94	9.3
49	107.4	2.37	8.3
50	128.6	2.10	15.4
51	124.6	2.29	9.2
52	127.2	2.36	10.2

Hill, C., W. Griffiths, and G. Judge. *Undergraduate Econometrics.* John Wiley & Sons, Inc., New York, NY 1997.

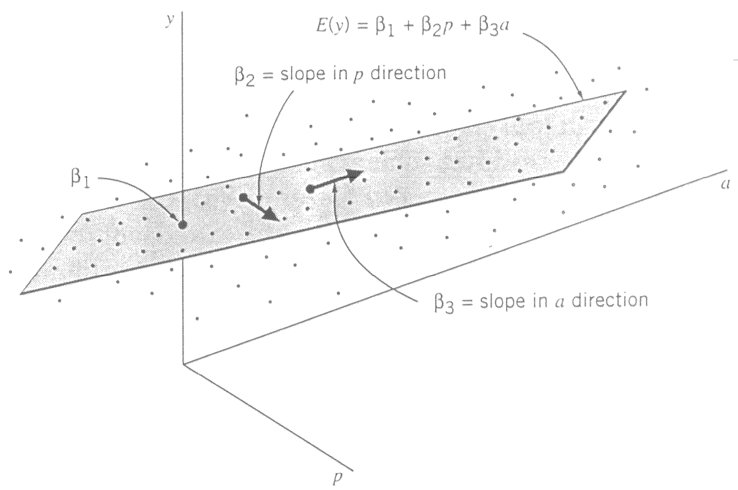

FIGURE ***7.1*** The multiple regression plane

Hill, C., W. Griffiths, and G. Judge. *Undergraduate Econometrics.* John Wiley & Sons, Inc., New York, NY 1997.

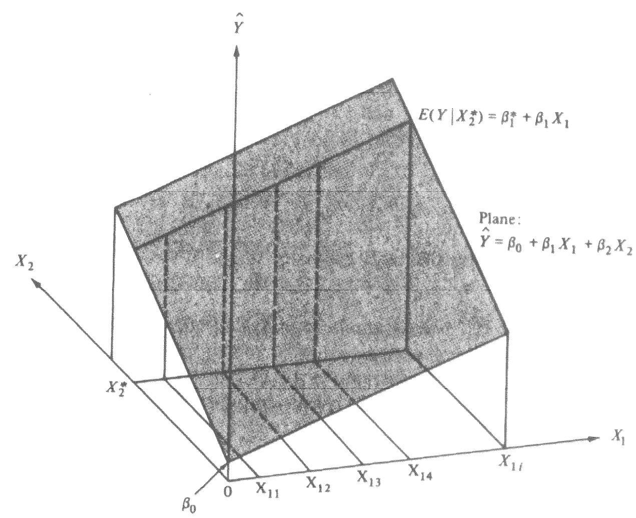

Figure 4.2 Typical Regression Plane for $K = 2$.

Johnson, Aaron C. Jr., Marvin B. Johnson, Rueben C. Buse. Econometrics: Basic and Applied. MacMillan Publishing Company. New York.1989.

Motivation for adding the error term is the same as for the simple linear regression model

Combined effect of other influences

- In reality, a large number of independent variables in addition to price and advertising affect total revenue

- Assume the other independent variables are unobservable, or we would include them in economic model

Approximation error

- Linear form of model may only be an approximation of the true relationship between total revenue, price and advertising

Random component of human behavior

- Knowledge of all variables that influence an weekly total revenue may not be sufficient to exactly explain observed revenue

To complete the statistical model, we must specify the assumptions about the error term,

- $E(e_t) = 0$

- $\text{var}(e_t) = E[e_t - E(e_t)]^2 = E[e_t^2] = \sigma^2$, or the error term is homoscedastic

- e_t are independent so that $Cov(e_t e_s) = 0$ for all $t \neq s$

- e_t follows a normal distribution

This can be summarized using the following notation,

$$e_t \sim N(0, \sigma^2) \qquad t = 1, \ldots, T$$

As we saw before, this is referred to as the *iid* normality assumption, which is shorthand for identical, independently distributed normal random variables

Now, let's explore some of the implications of the statistical model

Writing the statistical model as,

$$y_t = E[y_t] + e_t$$

Allows us to think of observed total revenue as consisting of two components,

- $E[y_t]$: expected, or mean, total revenue

- e_t: a random component that is unique to each week

We generated the same interpretation in the constant mean model and the simple linear regression model!

The mean component varies linearly with the level of price and advertising

$$E(y_t) = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t}$$

In other words, the average value of total revenue varies linearly with price and advertising

Now, consider the variance of total revenue

$$\text{var}[y_t] = E\left[(y_t - E[y_t])^2\right]$$

$$\text{var}[y_t] = E\left[(y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t})^2\right]$$

$$\text{var}[y_t] = E\left[e_t^2\right] = \sigma^2$$

This result is equivalent to saying that the variance of total revenue is not related to the level of price or advertising

Next, consider the covariance of total revenue between two values y_t and y_s,

$$\text{cov}[y_t, y_s] = E\left[(y_t - E[y_t])(y_s - E[y_s])\right]$$

$$\text{cov}[y_t, y_s] = E[(y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t}) \cdot (y_s - \beta_1 - \beta_2 x_{2,s} - \beta_3 x_{3,s})]$$

$$\text{cov}[y_t, y_s] = E\left[e_t e_s\right] = 0 \qquad t \neq s$$

Hence, if the errors are independent, then the selection of one week's observation does not influence whether another will be selected

We can now re-state the statistical model,

$$y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + e_t$$

where

- $E[e_t] = 0$
- $E[y_t] = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t}$
- $\text{var}[e_t] = \text{var}[y_t] = \sigma^2$
- $\text{cov}[e_t, e_s] = \text{cov}[y_t, y_s] = 0 \qquad t \neq s$

These assumptions can be written more compactly by noting that e_t and y_t are *iid* with the following distributions,

$$e_t \sim N(0, \sigma^2) \quad \text{and} \quad y_t \sim N(\beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t}, \sigma^2)$$

As in the case of simple regression, we also assume that,

- The independent variables are fixed and non-stochastic (fixed in repeated sampling)

- Same as saying *x's* are not random variables

A new assumption about the independent variables must be added in the case of multiple regression

- One x is not an exact linear function of the other x, and vice versa

- Or, it is not possible to write

$$x_{2,t} = m_1 + m_2 x_{3,t}$$

  for at least one non-zero value of m_1 or m_2

- Equivalent to assuming one of the explanatory variables has additional informational content not contained in the other explanatory variable

Assumptions of the Multiple Regression Model

MR1. $y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + e_t, \quad t = 1,\ldots,T$

MR2. $E(e_t) = 0 \Leftrightarrow E(y_t) = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t}$

MR3. $\text{var}(e_t) = \text{var}(y_t) = \sigma^2$

MR4. $\text{cov}[e_t, e_s] = \text{cov}[y_t, y_s] = 0 \qquad t \neq s$

MR5. The values of $x_{2,t}$ and $x_{3,t}$ are not random or exact linear functions of one another

MR6. $e_t \sim N(0,\sigma^2) \Leftrightarrow y_t \sim N(\beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + e_t, \sigma^2)$

Estimating the Parameters for the Statistical Model of Bay Area Rapid Food Total Revenue

The statistical model "explains" how the sample of total revenue data is generated

The problem at hand is how to use the sample information to estimate the unknown parameters

Just as before, we need a rule to systematically estimate β_1, β_2 and β_3 based on the observed sample of data

- The principle of least squared distance can again be used to find the desired estimates

- Minimize the sum of squares of the vertical distances between the plane and the sample observations

To begin the formal derivation, let's restate the statistical model,

$$y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + e_t$$

which can be re-written as,

$$e_t = y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t}$$

Then, given the sample observations on y and the x's, our objective is to minimize the following function,

$$S(\beta_1, \beta_2, \beta_3) = \sum_{t=1}^{T} e_t^{\,2} = \sum_{t=1}^{T} (y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t})^2$$

Since the values for y_t are known, S is solely a function of the unknown parameters β_1, β_2 and β_3

This function is a quadratic in terms of the unknown parameters β_1, β_2 and β_3

$\Rightarrow$ "bowl-shaped" function in 4-D!

Minimum value of function is found by taking the partial differentials of S with respect to β_1, β_2 and β_3,

$$\frac{\partial S}{\partial \beta_1} = 2\sum_{i=1}^{n}(y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t})(-1)$$

$$\frac{\partial S}{\partial \beta_2} = 2\sum_{t=1}^{T}(y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t})(-x_{2,t})$$

$$\frac{\partial S}{\partial \beta_3} = 2\sum_{t=1}^{T}(y_t - \beta_1 - \beta_2 x_{2,t} - \beta_3 x_{3,t})(-x_{3,t})$$

The values of β_1, β_2 and β_3 that make the partial derivatives equal zero are the least squares estimates, which are denoted b_1, b_2, and b_3

$$2\sum_{i=1}^{n}(y_t - b_1 - b_2 x_{2,t} - b_3 x_{3,t})(-1) = 0$$

$$2\sum_{t=1}^{T}(y_t - b_1 - b_2 x_{2,t} - b_3 x_{3,t})(-x_{2,t}) = 0$$

$$2\sum_{t=1}^{T}(y_t - b_{11} - b_2 x_{2,t} - b_3 x_{3,t})(-x_{3,t}) = 0$$

With a little more re-arranging, we can arrive at the following equations:

$$\sum_{t=1}^{T} y_t = Tb_1 + b_2 \sum_{t=1}^{T} x_{2,t} + b_3 \sum_{t=1}^{T} x_{3,t}$$

$$\sum_{t=1}^{T} y_t x_{2,t} = b_1 \sum_{t=1}^{T} x_{2,t} + b_2 \sum_{t=1}^{T} x_{2,t}^2 + b_3 \sum_{t=1}^{T} x_{2,t} x_{3,t}$$

$$\sum_{t=1}^{T} y_t x_{3,t} = b_1 \sum_{t=1}^{T} x_{3,t} + b_2 \sum_{t=1}^{T} x_{2,t} x_{3,t} + b_3 \sum_{t=1}^{T} x_{3,t}^2$$

These three equations are the normal equations in the three-variable least squares regression

Now, we have three unknowns and three equations, so we can solve for b_1, b_2, and b_3

The solution for the intercept should be familiar,

$$b_1 = \frac{1}{T} \sum_{t=1}^{T} y_t - b_2 \frac{1}{T} \sum_{t=1}^{T} x_{2,t} - b_3 \frac{1}{T} \sum_{t=1}^{T} x_{3,t}$$

$$= \bar{y} - b_2 \bar{x}_2 - b_3 \bar{x}_3$$

This result also proves an important fact about multiple regression

We can re-write the formula for b_1 as,

$$\bar{y} = b_1 + b_2\bar{x}_2 + b_3\bar{x}_3$$

which shows that the multiple regression plane must pass through the sample means of y, x_2 and x_3 (centroid)

The solutions for the two slope coefficients are much simplified if we use deviations from the mean notation where,

$$y'_t = y_t - \bar{y}$$
$$x'_{2,t} = x_{2,t} - \bar{x}_2$$
$$x'_{3,t} = x_{3,t} - \bar{x}_3$$

Then,

$$b_2 = \frac{\sum_{t=1}^{T} y'_t x'_{2,t} \sum_{t=1}^{T} x'^2_{3,t} - \sum_{t=1}^{T} y'_t x'_{3,t} \sum_{t=1}^{T} x'_{2,t} x'_{3,t}}{\sum_{t=1}^{T} x'^2_{2,t} \sum_{t=1}^{T} x'^2_{3,t} - \left(\sum_{t=1}^{T} x'_{2,t} x'_{3,t} \right)^2}$$

$$b_3 = \frac{\sum_{t=1}^{T} y'_t x'_{3,t} \sum_{t=1}^{T} x'^2_{2,t} - \sum_{t=1}^{T} y'_t x'_{2,t} \sum_{t=1}^{T} x'_{2,t} x'_{3,t}}{\sum_{t=1}^{T} x'^2_{2,t} \sum_{t=1}^{T} x'^2_{3,t} - \left(\sum_{t=1}^{T} x'_{2,t} x'_{3,t} \right)^2}$$

Three Notable Properties of LS Estimates

1. Sum of the estimated errors always equals zero

First, note that the estimated error for each observation is simply the actual observation on y minus the value projected by the estimated regression plane, or

$$\hat{e}_t = y_t - b_1 - b_2 x_{2,t} - b_3 x_{3,t}$$

Condition ["enforced"]() by the first normal equation

$$\sum_{t=1}^{T} (y_t - b_1 - b_2 x_{2,t} - b_3 x_{3,t}) = \sum_{t=1}^{T} \hat{e}_t = 0$$

1. Estimated regression plane must pass through the sample means of x_2, x_3 and y (centroid)

Shown by first noting that $b_1 = \bar{y} - b_2\bar{x}_2 - b_3\bar{x}_3$, which can be re-written as $\bar{y} = b_1 + b_2\bar{x}_2 + b_3\bar{x}_3$

2. Zero correlation between the estimated errors and x_2 or x_3, the explanatory variables

Condition "enforced" by the second and third normal equations

$$\sum_{t=1}^{T}(y_t - b_1 - b_2 x_{2,t} - b_3 x_{3,t})(x_{2,t}) = \sum_{t=1}^{T}\hat{e}_t x_{2,t} = 0$$

$$\sum_{t=1}^{T}(y_t - b_1 - b_2 x_{2,t} - b_3 x_{3,t})(x_{3,t}) = \sum_{t=1}^{T}\hat{e}_t x_{3,t} = 0$$

No tendency of estimated errors for observations above (below) the mean of x_2 or x_3 to be positive (negative) and *vice versa*

Sample Regression Output from Excel

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.93117
R Square	0.86708
Adjusted R Square	0.86166
Standard Error	6.06961
Observations	52

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	11776.1839	5888.0919	159.8280	0.0000
Residual	49	1805.1684	36.8402		
Total	51	13581.3523			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	104.7855	6.4827	16.1638	0.0000	91.7580	117.8130
X Variable 1	-6.6419	3.1912	-2.0813	0.0427	-13.0549	-0.2290
X Variable 2	2.9843	0.1669	17.8769	0.0000	2.6488	3.3198

Interpretation of Sample Estimates for Bay Area Rapid Food

$$\hat{y}_t = 104.785 - 6.642x_{2,t} + 2.984x_{3,t}$$

To interpret the coefficients note that total revenue and advertising are stated in units of $1,000

1. The coefficient on price $(x_{2,t})$ is negative and implies that an increase in price of $1 will lead to a fall in weekly revenue of $6,642. Or, stated positively, a reduction in price of $1 will lead to an increase in revenue of $6,642. Suggests that demand is price elastic.

2. The coefficient on advertising $(x_{3,t})$ is positive, and implies that an increase in advertising expenditure of $1,000 will lead to an increase in total revenue of $2,984.

3. The estimated intercept implies that if both price and advertising expenditure were zero, then total revenue earned would be $104,785. This is obviously not correct. The intercept is included in the model for mathematical completeness and to improve the model's predictive ability.

4. The estimated equation can also be used for prediction. Suppose management is interested in predicting total revenue for a price of $2 and an advertising expenditure of $10,000. This prediction is given by

$$\hat{y}_t = 104.785 - 6.642(2) + 2.984(10)$$
$$= 121.34$$

Thus, the predicted value of total revenue for the specified values of price and advertising is approximately $121,340.

A Word of Caution

The negative sign attached to price implies that reducing the price will increase total revenue

If taken literally, why should we not keep reducing the price to zero?

- Obviously this strategy will not continuously increase total revenue

- This makes the following important point: estimated regression models describe the relationship between the economic variables for values similar to those found in the sample data

- Extrapolating the results to extreme values is generally not a good idea

- In general, predicting the value of the dependent variable for values of the explanatory variables far from the sample values invites disaster