How to Hedge Selectively: A Bayesian Approach

by

Wei Shi and Scott H. Irwin[*]

Last update: November 22, 2008

---

[*]Walter Wei Shi is a senior associate at Morgan Stanley Investment Management, Global Risk & Analysis, and Scott H. Irwin is a professor at University of Illinois at Urbana-Champaign. Correspond to Wei Shi, 1221 6th Avenue, 34th Floor, New York, NY 10020. Phone: +212 762-6950. Fax: +212 507-8161. Email: wei.shi@morganstanley.com.

How to Hedge Selectively: A Bayesian Approach

Abstract

A Bayesian optimal hedging model is developed to quantitatively analyze selective hedging behavior. Numerical results confirm that subjective market views can have a substantial impact on a hedger's optimal position. The impact is most evident when the hedger speculates on future market price directions and/or is pessimistic about the effectiveness of hedging, i.e., a breakdown in the correlation among different markets. The results offer new insight into selective hedging behavior and contribute to explaining the large cross-sectional and time-series variation of hedging positions often observed in practice.

I.   Introduction

The theory of corporate risk management suggests that value-maximizing firms should hedge market price risk in order to reduce the variability of cash flows, and by doing so, reduce bankruptcy costs, tax payments, payments to stakeholders and costs of capital (Stulz, 1984; Smith and Stulz, 1985; Froot et al., 1993).[1] This theory predicts that the immediate objective of hedging is minimization of the variance of cash flows, and hence, risk management by itself should have zero net present value.

The actual practice of corporate risk management is seemingly at odds with the standard theory. Survey studies on risk management practice (e.g., Dolde, 1993; Bodnar et al., 1998; Geczy et al., 2005) report that corporate risk managers often have views on future market price directions and sometimes hedge selectively based on their views, a practice that has been termed "selective hedging" (Working, 1962; Stulz, 1996). For example, Bodnar et al. (1998) find that about one-half of the surveyed U.S. non-financial firms sometimes take active positions that reflect their views on future market price directions. Additional evidence on selective hedging is found in studies on risk management practices of gold mining firms (Tufano, 1996, 1998; Adam and Fernando, 2006; Brown et al., 2006), airline companies (Carter et al., 2002) and large U.S. non-financial firms (Beber and Fabbri, 2005). These studies report large cross-sectional and time-series variation in individual firms' derivative holdings.[2] The large variation is difficult to explain based on the variation in factors suggested by theory and indicates that a significant part of the variation may be

---

[1]The theory also suggests that managerial compensation contracts (i.e., agency problems) influence firms' risk management decisions (e.g., Stulz, 1984).

[2]For example, Beber and Fabbri (2005) report that managers adjust notional currency derivatives amounts in response to past currency returns and argue this is evidence of managers taking active views on currency movements. They also find that a measure of "speculative" intensity is related to personal characteristics of CEOs after controlling for other firm-related effects, a result that is inconsistent with derivatives use being driven by some type of optimal hedging policy.

attributed to selective hedging. Nonetheless, most of these studies also report that the gains from selective hedging are small and economically insignificant.

Further evidence on selective hedging has appeared in media reporting. For example, Placer Dome, Inc., a major Canadian gold producer, decided to completely abandon its policy of hedging gold price risk in 2000 because of a strong bullish view on future gold prices.[3] Recently, two budget airlines, Jet Blue and Southwest, are reported to have consistently profited from their hedging programs and hence improved their overall financial performance. In contrast, larger and more established but financially troubled airlines, which according to the theory should hedge more than the previous two firms, hedge little of their fuel price risk or do not hedge at all.[4]

In summary, there is widespread evidence that firms have goals other than variation-minimization in risk management. Specifically, some corporate risk managers attempt to profit from their hedging programs instead of merely minimizing risk.[5] The efficient market hypothesis suggests that current prices are the best predictors of future prices since all information has already been incorporated into current market prices, and consequently, it is impossible to consistently beat the market. However, many risk managers apparently believe they have the ability to outperform the market. Stulz (1996) argues that some firms may in fact have comparative advantages in information gathering and processing, and hence, outperform the market by hedging selectively. Geczy et al. (2005) recently present evidence on the characteristics of U.S. non-financial firms that "speculate" and the results are consistent with a belief on the part of firm managers that speculation (hedging with a view) is profitable due to information and cost advantages. Alternatively, selective

---

[3]See "Placer Dome alters hedging strategy, giving gold a lift," *Wall Street Journal*, February, 07, 2000.

[4]See "Hedge hog Southwest Air sharpens its teeth" and "For airlines, fuel hedging gets tricky," *Wall Street Journal*, May 19, 2005.

[5]Firms also may be "selective" in other aspects of corporate financial management. For example, Faulkender (2005) analyzes firms' selection of the interest rate exposure in their new debt issuances and suggests that interest rate risk management practices are primarily driven by speculation or myopia, not hedging considerations.

hedging may be due to decision-making biases that are well-documented in the behavioral finance literature (e.g., Debondt and Thaler, 1995; Shiller, 2003; Barberis and Thaler, 2003). For example, risk managers may be overconfident in their ability to outperform the market.

Despite the widespread use of selective hedging, relatively little research has attempted to explain how risk managers' subjective views influence hedging decisions. Noting that selective hedging can be understood as a special type of active asset allocation/portfolio optimization decision, Shi and Irwin (2005) develop an optimal hedging model based on the Bayesian portfolio optimization framework (e.g., Jorion, 1986; Frost and Savarino, 1986; Black and Litterman, 1992; Pástor, 2000). The Bayesian framework is a well-accepted paradigm for accommodating non-sample information, including subjective views, and parameter estimation risk within a portfolio optimization context. The Bayesian framework considers subjective views and parameter estimation risk together because model parameters are not directly observable and have to be estimated. In other words, model parameters are subject to both estimation errors and and investors' speculation. In an earlier study, Lence and Hayes (1994) also develop an optimal hedging model based on the Bayesian framework but the focus of their study is on parameter estimation risk.

Shi and Irwin (2005) limit consideration of subjective views and estimation risk to the expectation vector of asset returns, and thereby ignore the same concerns about the covariance matrix of asset returns. Subjective views and estimation risk regarding the covariance matrix are also important within an optimal hedging context. First, the optimal hedging position, particularly its pure hedging component, is sensitive to estimation errors in volatilities and correlations.[6] Second, the covariance matrix is difficult to estimate accurately in practice because it is conditional on the

---

[6]To see this point, consider the minimum-variance hedging ratio within a static one-period futures hedging context. The ratio, under certain assumptions, is equal to the ratio of the covariance between futures and cash price changes and the volatility of futures price changes. This ratio clearly is sensitive to subjective views and/or estimation errors in either component of the ratio.

available information set and may be subject to regime shifts. Finally, given the plethora of methods and data available for estimating the covariance matrix, individual risk managers may obtain forecasts of volatilities and correlations quite different from the market consensus.

In a separate vein, Brown and Khokher (2005) also develop a theoretical model for analyzing corporate risk management decisions when managers have a directional prediction on future price levels. They find that the optimal hedging strategy with 'a view' retains a partial exposure and requires rebalancing, a pattern that can help explain the large variation in hedging ratios mentioned earlier. Nonetheless, their model is not based on the Bayesian portfolio optimization framework. Given the portfolio interpretation of hedging decisions and the inherent link between subjective views and parameter estimation risk, we argue that subjective views and parameter estimation risk should be considered within a Bayesian framework.

In this study, we develop a new Bayesian optimal hedging model to quantitatively analyze selective hedging behavior. The model incorporates hedgers' subjective views and estimation risk regarding both the expectation vector and the covariance matrix of the price changes of the assets involved in hedging. An "empirical" Bayesian approach is adopted to allow a hedger to express his/her view(s) in a flexible and realistic manner. The new model not only provides researchers with a conceptual framework to quantitatively analyze the impact on hedging positions of subjective views, but also provides practitioners with a tool to adjust hedging positions consistent with their views regarding future market price movements.

We illustrate the impact of subjective views on hedging decisions (and how to hedge selectively) using four applications to an airline interested in hedging jet fuel price exposure. The numerical results confirm that subjective views can have a substantial impact on a firm's optimal position. The impact is most evident when the risk manager speculates on future market price

4

directions and/or is pessimistic about the effectiveness of hedging, i.e., a breakdown in the correlation among different markets. These results offer new insight into the influence of subjective views on hedging behavior and contribute to explaining the large cross-sectional and time-series variation of hedging ratios often observed in practice.

The rest of the paper is organized as follows. In next section, we develop the new Bayesian optimal hedging model. In the following section, we apply the Bayesian model to analyze the impact of subjective views and parameter estimation risk on an airline's optimal position under different scenarios. In the last section, we draw conclusions.

II. Theoretical Model

In this section, we first review the standard optimal hedging model and its typical implementation, and then derive the new Bayesian optimal hedging model based on the Bayesian portfolio optimization framework.

A. Standard Optimal Hedging Model

We begin by considering the standard one-period static optimal futures hedging model.[7] Some simplifying assumptions are made for ease of exposition and explicitness of results. Specifically, we assume: (1) a hedger has a fixed long position in a spot market and hedges the spot position using futures contracts matching the hedging horizon, and (2) the futures market is frictionless, i.e., there is no commissions, no margin requirements and no lumpiness, and (3) the hedger's objective

---

[7]See Anderson and Danthine (1980) and Myers and Thompson (1989) for a thorough analysis of this model.

is mean-variance maximization of the end-of-period profit/loss.[8] In addition, we assume that spot

and futures price changes follow a multivariate normal distribution,

(1)
$$\mu = \begin{bmatrix} \mu_s \\ \mu_f \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \sigma_s^2 & \sigma_{sf} \\ \sigma_{fs} & \sigma_f^2 \end{bmatrix}$$

where μ and Σ are the expectation vector and the covariance matrix of price changes, respectively.

Subscripts s and f denote spot and futures, respectively. Then, the mean-variance maximization

of the hedger's end-period profit/loss is,

(2)
$$\max_{Y_f} \begin{bmatrix} Y_s & Y_f \end{bmatrix} \left(\begin{bmatrix} \mu_s \\ \mu_f \end{bmatrix} - \frac{\tau_a}{2} \begin{bmatrix} \sigma_s^2 & \sigma_{sf} \\ \sigma_{fs} & \sigma_f^2 \end{bmatrix} \right)$$

where τ_a is the absolute Arrow-Pratt risk aversion coefficient and Y_s and Y_f are the hedger's spot

and futures positions, respectively. The optimal hedging position is determined via the first order

condition (FOC) of the mean-variance maximization,

(3)
$$Y_f^* = \frac{\mu_f}{\tau_a \sigma_f^2} - Y_s \frac{\sigma_{sf}}{\sigma_f^2}$$

where the first and second terms are, respectively, the speculative and the hedging components of

the optimal position. In practice, the model is often implemented with the Parameter Certainty

Equivalent (PCE) procedure, which requires a hedger to directly substitute estimates for the true

---

[8]Because the focus in this study is the optimal hedging decision of a risk manager responsible for managing
price risk in a particular market, the objective of hedging is assumed to be mean-variance maximization, whereas,
in corporate risk management theory, the objective of hedging is frequently assumed to be maximization of a firm's
expected market value or managerial expected utility. The former objective concerns risk managers' individual hedging
decisions while the latter concerns corporate hedging policies/strategies at a higher level of corporate management.

but unknown parameters. The optimal hedging position according to the PCE procedure is

$$(4) \qquad \hat{Y}_f^* = \frac{\hat{\mu}_f}{\tau_a \hat{\sigma}_f^2} - Y_s \frac{\hat{\sigma}_{sf}}{\hat{\sigma}_f^2}$$

where $\hat{\mu}_f, \hat{\sigma}_f^2$ and $\hat{\sigma}_{sf}$ are, respectively, the estimates for μ_f, σ_f^2 and σ_{sf}.

B. Bayesian Model

With the Bayesian portfolio optimization framework applied to hedging, the optimal position is determined via mean-variance maximization conditioned on the predictive expectation vector and covariance matrix of asset price changes. The Bayesian framework can be implemented with either a "pure" Bayesian approach or an "empirical" Bayesian approach. With a "pure" Bayesian approach, the prior distribution is calibrated with non-sample information, including subjective views and updated with sample data via Bayesian updating. In practice, hedgers are unlikely to have subjective views on more than one or two parameters of the prior distribution or only the relative relation of the parameters of the prior distribution; therefore, they are unlikely to be able to calibrate the entire prior distribution using only non-sample information. Consequently, a pure Bayesian approach imposes restrictive and unrealistic assumptions regarding calibration of the prior distribution. In comparison, an empirical Bayesian approach calibrates the prior distribution using sample data, which should contain enough information to estimate all the parameters of the prior distribution. Additionally, an empirical Bayesian approach allows hedgers to express their views in a flexible and realistic manner (Black and Litterman, 1992)

To accommodate estimation risk in both the expectation vector and covariance matrix, we specify the prior distributions for the expectation vector and covariance matrix as a normal prior dis-

tribution and an inverse-Wishart distribution, respectively. The prior distributions are conveniently

parameterized in term of hyperparameters as,

$$(5) \qquad \mu \sim \mathbb{N}(\mu_0, \kappa_0^{-1}\Sigma_0) \text{ and } \Sigma \sim \mathbf{W}_{\nu_0}^{-1}(\Sigma_0)$$

where the expectation vector, μ, is assumed to follow a multivariate normal distribution denoted

as $\mathbb{N}(\mu_0, \kappa_0^{-1}\Sigma_0)$, with μ_0 the prior belief concerning the expectation vector and $\kappa_0^{-1}\Sigma_0$ the confi-

dence level associated with that belief. Similarly, the covariance matrix, Σ, is assumed to follow

an inverse-Wishart distribution denoted as $\mathbf{W}_{\nu_0}^{-1}(\Sigma_0)$, where Σ_0 is the prior belief concerning the

covariance matrix and ν_0 measures the confidence level associated with that belief.[9]

A hedger's subjective views are classified into two categories: views concerning the expectation

vector and views concerning the covariance matrix. We follow Black and Litterman (1992) in

expressing views concerning the expectation vector as a normal distribution. Specifically,

$$(6) \qquad \mathbf{P}\mu \sim \mathbb{N}_k(\mathbf{q}, \Omega)$$

where $\mathbf{P}$ denotes a $k \times n$ "weight" matrix, each row of which, designates the weights of the assets

in a "view portfolio" and $\mathbf{q}$ is a $k \times 1$ vector, each entry of which specifies the expected return of

the "view" portfolio and Ω is a $k \times k$ diagonal matrix where the ith diagonal element is $\kappa_{1,i}^{-1}\mathbf{p}_i\Sigma_0\mathbf{p}_i'$,

---

[9]Note that the prior distributions for the expectation vector and covariance matrix are specified independently, that is, the distribution of μ does not depend on Σ. This prior specification allows an analytic solution to the optimal hedging problem to be derived when an empirical Bayesian approach is used. An alternative choice to accommodate estimation risk in both the expectation vector and covariance matrix is a normal-inverse-Wishart prior distribution (Lence and Hayes, 1994), which, however, does not allows an analytic solution if an empirical Bayesian approach is adopted. Moreover, a normal-inverse-Wishart prior distribution forces dependence between the distribution of the expectation vector and that of the covariance matrix. This assumption is unnecessary and restrictive since an expert's opinion concerning the expectation vector generally is little influenced by his/her views concerning the covariance matrix, and vice versa(e.g., Garthwaite and Al-Awadhi, 2001).

which measures the confidence level of the ith subjective view and where $\mathbf{p}_i$ denotes the ith row of the weight matrix. Notice that we assume that the number of the views does not exceed the number of assets and subjective views are independent from one another. Subjective views concerning the covariance matrix are expressed as an inverse-Wishart distribution,

$$\Sigma \sim \mathbf{W}_{v_1}^{-1}(\Sigma_1) \tag{7}$$

where Σ_1 and v_1 denote, respectively, the view concerning the covariance matrix and the confidence level associated with the view.

The prior distributions and subjective views are combined via Bayesian updating. The resulting predictive distribution is a multivariate t-distribution with the following expectation vector and covariance matrix (Scherer, 2004, p.113-115),

$$
\begin{aligned}
\mu_2 &= \left[(\kappa_0^{-1}\Sigma_0)^{-1} + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\left[(\kappa_0^{-1}\Sigma_0)^{-1}\mu_0 + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{q}\right] \\
&= \left[(\kappa_0^{-1}\Sigma_0)^{-1} + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1}\left[(\kappa_0^{-1}\Sigma_0)^{-1}\mu_0 + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P}\cdot\mu_1\right], \\
\Sigma_2 &= \frac{v_0 + v_1 + 1}{v_0 + v_1 - d - 2}\left(\frac{v_0}{v_0 + v_1}\Sigma_0 + \frac{v_1}{v_0 + v_1}\Sigma_1\right),
\end{aligned}
\tag{8}
$$

where μ_2 and Σ_2 denote the predictive expectation vector and covariance matrix, respectively, d denotes the number of assets, and μ_1 denotes $\mathbf{P}'(\mathbf{P}\mathbf{P}')^{-1}\mathbf{q}$. The predictive expectation vector is a weighted average of the prior information and the view concerning the expectation vector. Similarly, the predictive covariance matrix is a scaled weighted average of the prior and the view concerning the covariance matrix.[10]

---

[10]The predictive distribution is derived based on the derivations in Black and Litterman (1992) and Scherer (2004, p.113-115). Detailed derivation of the theoretical model in this section and the model's applications in the next section are available from the authors upon request.

Finally, we can plug the elements of the predictive distribution into the standard optimal hedging model to determine the optimal hedging position. With a one-period static futures hedging setup, the optimal position according to the Bayesian procedure is,[11]

$$\hat{Y}_f^* = \frac{\mu_{f2}}{\tau_a \cdot \sigma_{f2}^2} - Y_s \frac{\sigma_{sf2}}{\sigma_{f2}^2}$$

(9)

where the subscript 2 denotes the predictive distribution (equation 8). The formula shows that with the Bayesian procedure, one should determine his/her optimal position by plugging in the elements of the predictive distribution, which combines information from the prior (sample data) and subjective views (non-sample information) together via Bayesian updating.

The Bayesian procedure incorporates the PCE (Parameter Certainty Equivalence) procedure as a special case when the hedger has no confidence about his/her subjective views. As $\kappa_{1,i} \longrightarrow 0$ and $v_1 \longrightarrow 0$, the Bayesian optimal position becomes,

$$\hat{Y}_f^* = \frac{\mu_{f0}}{\tau_a \cdot \frac{v_0+1}{v_0-4} \cdot \sigma_{f0}^2} - Y_s \frac{\sigma_{sf0}}{\sigma_{f0}^2}.$$

(10)

The formula shows that under the above assumptions the Bayesian position collapses to the PCE position with the exception of a factor of proportionality, $\frac{v_0-4}{v_0+1}$, in the speculative component. Note that the factor of proportionality approaches one as the confidence level on the prior approaches infinity, i.e., $v_0 \longrightarrow \infty$, and in this case, the difference between the Bayesian and PCE positions disappears. In reality, because of a finite sample size, a hedger would not put infinite confidence on the prior (sample data), and thus the factor of proportionality should always lie between 0 and

---

[11] The Bayesian model can be generalized to accommodate hedging with multiple assets, transaction costs, quantity risk and a multi-period setting.

1 and move closer to 1 as the sample size grows. Consequently, the speculative component of the Bayesian position is always smaller than that of the PCE position for any finite confidence level on the prior and the PCE position overstates the speculative component by the proportion $1 - \frac{v_0 - 4}{v_0 + 1}$.

The Bayesian procedure also incorporates the PPI (Perfect Parameter Information) procedure as a special case where a hedger puts infinite confidence on his/her subjective views. As $\kappa_{1,i} \longrightarrow \infty$ and $v_1 \longrightarrow \infty$, the Bayesian optimal position becomes,

$$(11) \qquad\qquad \hat{Y}_f^* = \frac{\mu_{f1}}{\tau_a \cdot \sigma_{f1}^2} - Y_s \frac{\sigma_{sf1}}{\sigma_{f1}^2}$$

The formula shows that the Bayesian position collapses to the PPI position as the confidence on the subjective views approaches infinity.

The Bayesian procedure not only nests the PCE and PPI procedures as two extreme cases, but also provides an optimal way to combine information from the prior (sample data) and subjective view (non-sample information) when the hedger is neither absolutely confident about the prior nor about his/her subjective views. In a non-extreme case, the hedger's optimal position according to the Bayesian procedure lies between the PCE and PPI results and tilts toward either side in response to the magnitude of the deviation of the views from the prior and the confidence levels assigned to those views.

C. Calibration of the Bayesian Model

To implement the Bayesian model, one needs a method to calibrate the confidence levels for the prior distribution. The assignment of confidence levels for the prior is not an easy task (e.g., Bevan and Winkelmann, 1998; He and Litterman, 1999; Fusal and Meucci, 2003). For simplicity,

we set both κ_0 and ν_0 equal to the number of sample observations. The procedure is valid if sample observations follow IID normal distribution. Given widespread evidence that asset returns do not follow IID normal distributions, one may choose more sophisticated calibration methods. Nevertheless, the procedure can still serve as a benchmark in assigning confidence levels.

One also needs a tool to gauge the deviation of subjective views from the prior distribution (i.e., sample data). Because the Bayesian factor is a well developed and frequently used tool for model comparison and selection (e.g., Bernardo and Smith, 1994; O'Hagan et al., 2004), it is used to measure the relative probability of the posterior distribution conditional on the prior distribution.[12] More specifically, we compute the "Weight of Evidence" (WoE), which is the logarithm of the Bayesian factor,

$$
\begin{aligned}
\ln\left(BF_{2,0}(\mathbf{I}_t)\right) = & \frac{p(\mathbf{I}_t|\mu_2, \boldsymbol{\Sigma}_2)}{p(\mathbf{I}_t|\mu_0, \boldsymbol{\Sigma}_0)} \\
= & -\frac{\kappa_0}{2}\ln\frac{|\boldsymbol{\Sigma}_2|}{|\boldsymbol{\Sigma}_0|} + \frac{\kappa_0}{2} \times d - \frac{\kappa_0}{2}\text{tr}(\boldsymbol{\Sigma_0}\boldsymbol{\Sigma}_2^{-1}) - \frac{\kappa_0}{2}(\mu_0 - \mu_2)'\boldsymbol{\Sigma}_2^{-1}(\mu_0 - \mu_2)
\end{aligned}
$$
(12)

where $BF_{2,0}(I_t)$ denotes the Bayesian factor in favor of the posterior distribution and against the prior distribution. Generally speaking, Bayesian decision theory (e.g., Bernardo and Smith, 1994, pp.389-395) shows that a WoE greater than zero signifies that the posterior distribution is more plausible in the light of sample data, while a WoE smaller than zero signifies that the prior distribution is more plausible in the light of sample data. Because we estimate μ_0 and $\boldsymbol{\Sigma}_0$ by maximizing the likelihood function $p(\mathbf{I}_t|\mu, \boldsymbol{\Sigma})$, the denominator of the Bayesian factor, $p(\mathbf{I}_t|\mu_0, \boldsymbol{\Sigma}_0)$, should be considered the unrestricted optimum while the numerator of the Bayesian factor, $p(\mathbf{I}_t|\mu_2, \boldsymbol{\Sigma}_2)$, should be considered the restricted optimum. As a result, WoE should always be below zero and

---

[12]See Harvey et al. (2003) for an application of the Bayesian factor in a portfolio optimization context. See also Bevan and Winkelmann (1998), He and Litterman (1999) and Fusal and Meucci (2003) for alternative calibration methods.

doubt is cast on the views if the WoE is too small.

Note that the Bayesian factor shown above is akin to the widely-used Likelihood ratio test statistic (Greene, 2002, p.152). Specifically, the large-sample distribution of $-2\ln\left(BF_{2,0}(\mathbf{I}_t)\right)$ should follow a χ^2 distribution with the degrees of freedom equal to the number of restrictions imposed. Because the hedging decision involves 2 assets in this study, there are 2 restrictions imposed on the expectation vector and 3 on the covariance matrix and hence the total number of restrictions and the degrees of freedom equal 5. The p-value of $-2\ln\left(BF_{2,0}(\mathbf{I}_t)\right)$ is used in the following section to signify the deviation of subjective views from the prior distribution.

III. Applications

In this section, we apply the new Bayesian model to analyze the impact on a hedger's optimal position of subjective views and parameter estimation risk under different scenarios.

A. Optimal Hedging with a Directional View

In the first application, we analyze how a hedger' view about the price trend in a futures market could influence his/her optimal position. Although the unbiasedness hypothesis of futures prices is often supported by empirical studies (e.g., Fama and French, 1987), as noted in the introduction, practitioners sometimes still have views about the direction of futures prices and may actively take positions based on their views. This type of view scenario is similar to that considered by Shi and Irwin (2005), except their model ignores estimation risk concerning the covariance matrix.

Assume that a hedger has no view concerning the covariance matrix, signified by $v_1 = 0$, yet has an opinion concerning the direction of the futures price change over the hedging horizon. The

13

view can be conveniently expressed using equation (6) as,

$$(13) \qquad \mathbf{P} = \begin{bmatrix} 0 & 1 \end{bmatrix} \text{ and } q = \begin{bmatrix} q \end{bmatrix}$$

Using the Bayesian model (equation 9), the optimal hedging position with the directional view is,

$$(14) \qquad \hat{Y}_f^* = \frac{\mu_{f0} + \frac{\kappa_1}{\kappa_0 + \kappa_1}(q - \mu_{f0})}{\tau_a \cdot \frac{v_0 + 1}{v_0 - 4} \cdot \sigma_{f0}^2} - Y_s \cdot \frac{\sigma_{sf0}}{\sigma_{f0}^2}.$$

The equation shows that a directional view has an impact on the speculative component but not on the hedging component of the optimal position.[13] The impact is determined by the magnitude of the view, $q - \mu_{f0}$, and the relative confidence level of the view, κ_1 v.s. κ_0. As the magnitude and/or the confidence level of the view decreases and approaches zero, the Bayesian position tilts toward and eventually collapses into the PCE position. However, if the magnitude of the directional view is significant and the hedger is quite confident about the view, the view could have a substantial impact on the optimal position.

We further illustrate the impact of a directional view on the optimal hedging position using a numerical example. For simplicity and brevity, we adopt a textbook example from Hull (2002, p.80-82) with minor modifications. The example assumes that an airline expects to purchase two million gallons of jet fuel in one month and decides to use heating oil futures for hedging. The estimated standard deviations of monthly changes in spot jet fuel prices and in heating oil futures prices are, respectively, \$0.0263/gallon and \$0.0313/gallon. The estimated correlation coefficient between the two is 0.9284; and the estimated expected monthly changes in spot jet fuel prices and

---

[13]The hedging component is the same as with the PCE procedure because the elements of the predictive covariance matrix change proportionately.

in heating oil futures prices are, respectively, $\$-0.0009$/gallon and $\$-0.0002$/gallon. Note that the sample averages, standard deviations and correlation are the same as in the textbook example. The prior distribution is calibrated with sample estimates and the confidence levels for the prior are assigned as $\kappa_0 = 15$ and $\nu_0 = 15$, equal to the number of sample observations. To compute the speculative component of a hedger's optimal hedging position, we need to know the absolute risk-aversion (ARA) coefficient for the hedger. We use "risk premiums" (Babcock et al., 1993, eq. 4) to compute reasonable ARAs. If the appropriate risk premium is either 10% or 25% or 50% of the standard deviation for revenue of the "naked" spot position, then the corresponding ARA is either 0.00000063, 0.00000164 or 0.00000382.

The hedger is assumed to have either a bullish or bearish directional view. A bullish (bearish) view means that the hedger speculates that the expected change in futures prices will be $\$+0.03$/gallon ($\$-0.03$/gallon) instead of $\$-0.0009$/gallon according to the sample data (the market consensus). A view is assumed to be associated with either a high confidence level ($\kappa_1 = 10$) or a low confidence level ($\kappa_1 = 5$). Under an IID distribution assumption, the high (low) confidence level means that the hedger's confidence in the view is equivalent to using 10 (5) sample observations in estimation. A no view scenario is considered for comparison. In addition, the hedger is assumed to have no view regarding the covariance matrix, signified by $\nu_1 = 0$.

[INSERT TABLE 1 HERE]

The variation of the optimal position in response to directional views is reported in table 1, which shows that a directional view could have a substantial impact on a hedger's speculative component and hence his/her overall optimal position. For example, with a 25% risk premium assumed, a bullish directional view associated with a high confidence level leads to speculative,

hedging and overall optimal positions of $964,198$ gallons, $1,560,187$ gallons and $2,524,384$ gallons, respectively; while a bearish directional view associated with a high confidence level results in values for the same components of $-1,055,065$ gallons, $1,560,187$ gallons and $505,122$ gallons, respectively. These can be compared to the relevant "no view" speculative, hedging and overall optimal positions of $-75,722$ gallons, $1,560,187$ gallons and $1,484,464$ gallons, respectively. The comparisons show that the impact of a directional view is limited to the speculative component, which varies in sign and magnitude according to the direction and confidence level of the view. The p-values of the WoE's suggest that most of the directional view scenarios are not extreme from the perspective of the prior distribution. For example, the p-value is 0.80 for a bullish view with a high confidence level, where the hedger expects the change in futures price to be \$0.03/gallon instead of \$ -0.0009/gallon for the prior. The results also show the expected pattern that increasing the risk premium has a dampening influence on the hedger's risk taking. All else equal, as the risk premium increases, the impact of the view on a hedger's speculative component, and thus the overall position, decreases.

In summary, the results show that a directional view can have a substantial impact on a hedger's optimal position. With a moderate level of risk-aversion (25% risk premium), a directional view may change the optimal position from 34% to 170% of the "no view" position. The specific impact depends on the magnitude and/or the confidence level of the view. The substantial variation of optimal positions in response to the different directional views may help to explain the well-documented phenomenon of wide cross-sectional and time-series variation of hedging positions (ratios) observed in practice.

B. Optimal Hedging with a View on the Covariance Matrix

Given widespread evidence on the time-varying nature of conditional volatility and regime switch-ing, some hedgers may have views concerning the volatilities and correlations of the different markets involved in hedging decisions. While a hedger may use GARCH-type models or regime-switching models to estimate time-varying hedging position (ratios), the Bayesian model provides a formal method for analyzing the impact on optimal positions of this type of view. For example, a hedger may be quite skeptical about the effectiveness of futures hedging. That is, the hedger is quite pessimistic about the accuracy of the covariance matrix estimated from historical data. The hedger may conjecture that the volatilities of both spot and futures markets are under-estimated by the market consensus based on sample data, and/or the correlation between the two markets is over-estimated by the market consensus. Consequently, the hedger may formulate an alternative estimate of the covariance matrix denoted as Σ_2.

The hedger in this scenario is assumed to have no view regarding the expectation vector, signi-fied by $\kappa_1 = 0$. Using the Bayesian model above (equation 9), the optimal hedging position with a view on the covariance matrix is,

$$(15) \qquad \hat{Y}_f^* = \frac{\mu_{f0}}{\tau_a \cdot \frac{v_0 + v_1 + 1}{v_0 + v_1 - 4} \cdot \left(\frac{v_0}{v_0 + v_1} \sigma_{f0}^2 + \frac{v_1}{v_0 + v_1} \sigma_{f1}^2 \right)} - Y_s \cdot \frac{\frac{v_0}{v_0 + v_1} \sigma_{sf0} + \frac{v_1}{v_0 + v_1} \sigma_{sf1}}{\frac{v_0}{v_0 + v_1} \sigma_{f0}^2 + \frac{v_1}{v_0 + v_1} \sigma_{f1}^2}.$$

The equation suggests that a view on the covariance matrix has an impact on both the speculative and hedging components of the optimal position. As in the first example, the Bayesian optimal position nests the PCE and PPI results as two extreme cases, tilting towards either side depending on the magnitude and relative confidence level of the view on the covariance matrix.

As before, we illustrate the variation of the optimal hedging position in response to views on

the covariance matrix with a numerical example. The airline hedging setting and prior calibration are the same as in the first example. In addition, we assume that the hedger suspects that the volatilities of the futures and spot markets will be twice that of the market consensus (the prior), and/or the correlation between the two markets will be three-quarters that of the market consensus. The confidence level of the view is again assigned as $v_1 = 10$ or $v_1 = 5$.

[INSERT TABLE 2 HERE]

The variation of the optimal position in response to the view concerning the covariance matrix is reported in table 2. The results suggest that a view concerning the covariance matrix will have a measurable impact on either the hedging or speculative component, and thus, the overall optimal position. For example, with a 25% risk premium assumed and no view, the speculative, hedging and overall optimal positions are $-75,722$ gallons, $1,560,187$ gallons and $1,484,464$ gallons, respectively. If a hedger takes a pessimistic view about hedging effectiveness; that is, the hedger suspects that volatilities will be twice that of the market consensus and correlation just three-quarters that of the market consensus, and has a high confidence level about the view, the speculative, hedging and overall optimal positions are $-40,437$ gallons, $1,276,516$, gallons and $1,236,080$ gallons, respectively. The reduction in the optimal position is primarily due to the decreased correlation expressed in the hedger's view on the covariance matrix, which reduces the hedging component. This can be easily seen in the scenarios where the hedger continues to believe volatilities will be twice that of the market consensus but has no view on correlation. In this case, the speculative, hedging and overall optimal positions are $-40,437$ gallons, $1,560,187$ gallons and $1,519,750$ gallons, respectively. The hedging component is the same as in the no view case and

the overall optimal position differs little from the no view scenario.[14] Overall, with a 25% risk premium assumed, a view on the covariance matrix can change the optimal position from 83% to 102% of the "no view" position.

This analysis suggests the sensible result that hedgers will hedge less when they have more doubt about the effectiveness of hedging, in particular, the correlation between price movement in the spot and futures markets. The p-values of WoE's indicate that most of view scenarios are not extreme except for the combination of increasing volatilities and decreasing correlation. Also notice that the relative risk-aversion coefficient has minimal impact on the speculative component and no impact on the hedging component. In the same vein as before, the view and uncertainty regarding covariance matrix estimates can help explain the wide cross-sectional and time-series variation in hedging ratios observed in practice.

C.   Optimal Hedging with a Combined View

It is possible that a hedger has a "combined view." That is, a combination of a directional view and a view on the covariance matrix. Using the Bayesian model (equation 9), the optimal hedging position with a combined view is,

$$(16) \qquad \hat{Y}_f^* = \frac{\mu_{f0} + \frac{\kappa_1}{\kappa_0+\kappa_1}(q - \mu_{f0})}{\tau_a \cdot \frac{v_0+v_1+1}{v_0+v_1-4} \cdot \left(\frac{v_0}{v_0+v_1}\sigma_{f0}^2 + \frac{v_1}{v_0+v_1}\sigma_{f1}^2\right)} - Y_s \cdot \frac{\frac{v_0}{v_0+v_1}\sigma_{sf0} + \frac{v_1}{v_0+v_1}\sigma_{sf1}}{\frac{v_0}{v_0+v_1}\sigma_{f0}^2 + \frac{v_1}{v_0+v_1}\sigma_{f1}^2}.$$

The equation is more complicated than those in the previous two examples due to the combination of a directional view and a view on the covariance matrix. Because the closed-form solution is difficult to interpret, we use our numerical example to illustrate the impact of a combined view on

---

[14]The view causes proportional changes in σ_{sf2} and σ_{f2}^2, which are, respectively, the numerator and the denominator of the hedging component (see equation 9). Consequently, the hedging component is the same as in the no view case.

the hedger's optimal position. The airline hedging setting and prior calibration are the same as in the first two examples. View scenarios from the previous two examples are combined. Results for a 25% risk premium are reported in table 3.[15] The results show that combining a directional view and a view on the covariance matrix, as would be expected, introduces even greater variation in the optimal hedging position. A combined view can change the optimal position from 11% to 171% of the "no view" position.

[**INSERT TABLE 3 HERE**]

D.   Optimal Hedging with a Basis View

In the next application, we analyze the impact on a hedger's optimal hedging position of a basis view, which is the hedger's view regarding the end-of-hedging-period difference between cash and futures prices. Shi and Irwin (2005) treat a basis view as a "relative" opinion concerning the performance of the spot and futures market, but assume that a basis view has no effect on a hedger's perception about the covariance matrix of asset price changes. The theory of storable commodities, which is applicable to the airline example considered here, predicts that as the basis strengthens (weakens) the volatilities of spot and futures markets may increase (decrease) and the correlation between them may decrease (increase) (French, 1986; Fama and French, 1987, 1988; Williams and Wright., 1991; Ng and Pirrong, 1994). For this reason, a basis view for a storable commodity should also be considered a type of "combined" view expressed on both the expectation vector and the covariance matrix.

---

[15]Due to space constraints, we only present results for the 25% risk premium. Results for the 10% and 50% risk premiums are available for authors upon request. Note that results presented in table 3 incorporate the results in tables 1 and 2 for a 25% risk premium as special cases where the hedger express a view on either the expectation vector or the covariance matrix but not on both.

Assume that at the beginning of the hedging period, a hedger speculates the the end-of-the-period basis should be $\tilde{b}_1$, while historical data project it at b_1. Part of the view concerns the relative performance of the spot and futures markets, which can be expressed using equation 6 as,

(17) $$\mathbf{p} = [1 \; -1] \text{ and } \mathbf{q} = \tilde{b}_1 - b_0$$

where b_0 denotes the basis observed at the beginning of the period. Note that $\kappa^{-1}\mathbf{P\Sigma P}' = \kappa^{-1}(\sigma_s^2 - 2\sigma_{sf} + \sigma_f^2)$. Moreover, based on his/her estimate of the end-of-period basis, the hedger may also form an opinion about the covariance matrix, which is expressed with equation 7 as $\mathbf{\Sigma} \sim \mathbf{W}_{\nu_1}^{-1}(\mathbf{\Sigma}_1)$. We can then plug in the elements of the predictive distribution into the standard optimal hedging model (equation 3) to derive the closed-form solution for the optimal hedging position,

(18) $$\hat{Y}_f^* = \frac{\mu_{f0} + \frac{\kappa_0\kappa_1}{\kappa_0\kappa_1+\kappa_0^2} \cdot \frac{\sigma_{s0}^2-\sigma_{sf0}}{\sigma_{s0}^2-2\sigma_{sf0}+\sigma_{f0}^2} \cdot (\tilde{b}_1 - b_1)}{\tau_a \cdot \frac{\nu_0+\nu_1+1}{\nu_0+\nu_1-4} \cdot \left(\frac{\nu_0}{\nu_0+\nu_1}\sigma_{f0}^2 + \frac{\nu_1}{\nu_0+\nu_1}\sigma_{f1}^2\right)} - Y_s \cdot \frac{\frac{\nu_0}{\nu_0+\nu_1}\sigma_{sf0} + \frac{\nu_1}{\nu_0+\nu_1}\sigma_{sf1}}{\frac{\nu_0}{\nu_0+\nu_1}\sigma_{f0}^2 + \frac{\nu_1}{\nu_0+\nu_1}\sigma_{f1}^2}.$$

Because the closed-form solution is again difficult to interpret, we focus on a numerical example to illustrate the impact of a basis view on the hedger's optimal position. The hedging setting and prior calibration are the same as in the previous examples. The market consensus is that basis will strengthen by \$0.0011/gallon, which is the difference between expected changes in spot and futures prices. However, the hedger has either a stronger or weaker basis view. A strong basis view means that the hedger suspects that basis will be strengthened by \$0.02/gallon and the volatilities of spot and futures markets should be twice that of the prior (the market consensus) and the correlation three-quarters of the prior. In contrast, a weaker basis view means that the hedger suspects that the basis will be \$ − 0.02/gallon, and volatilities of spot and futures half of the prior and the

correlation the same as that of the prior. The basis view is associated with either a high confidence level ($\kappa_1 = 10$ and $\nu_1 = 10$) or a low confidence level ($\kappa_1 = 5$ and $\nu_1 = 5$). A no view scenario is considered for comparison.

[**INSERT TABLE 4 HERE**]

The variation of the optimal position in response to the change in the basis view is reported in table 4. The results suggest that a basis view has a large impact on a hedger's optimal position. A stronger view induces the hedger to increase his/her short position in the speculative component, while a weaker view induces the hedger to switch from a short position to a long position in the speculative component. In contrast, regardless of direction, a basis view decreases the hedger's long position in the hedging component. With a 25% risk premium assumed and no view, the speculative, hedging and overall optimal positions are $-75,722$ gallons, $1,560,187$ gallons and $1,484,464$ gallons, respectively. A weaker basis view associated with a high confidence level can change the speculative, hedging and overall optimal positions to $1,763,615$ gallons, $1,560,187$ gallons and $3,323,802$ gallons, respectively; while a stronger basis view associated with a high confidence level can change them to $-522,790$ gallons, $1,276,516$ gallons, $753,726$ gallons, respectively. Overall, a basis view may change the optimal position from 51% to 224% of the "no view" position. A hedger's relative risk-aversion coefficient has an impact on the speculative component and no impact on the hedging component. Comparing the results in table 4 to those in tables 1 and 2 suggests that the impact of a basis view is similar to that of a directional view yet more substantial than that of a view on the covariance matrix.

IV. Conclusions

The theory of corporate risk management suggests that the rational motivation for hedging is minimization of the variance of cash flows, and hence, risk management by itself should have zero net present value. The actual practice of corporate risk management is seemingly at odds with the theory. Various survey, industry studies and media reports show that firms may have goals other than variation-minimization in risk management. More specifically, many corporate risk managers apparently attempt to profit by hedging selectively based on their views about future market price movements. The widespread use of selective hedging may account for much of the large cross-sectional and time-series variation in individual firms' hedging ratios often observed in corporate risk management practice. This large variation is difficult to explain based on variation in factors suggested by the theory of corporate risk management.

Despite the widespread use of selective hedging, relatively little research has attempted to explain how risk managers' subjective views influence hedging decisions. In this study, we develop a new Bayesian optimal hedging model to quantitatively analyze selective hedging behavior. The model incorporates hedgers' subjective views and estimation risk regarding both the expectation vector and the covariance matrix of the price changes of the assets involved in hedging. An "empirical" Bayesian approach is adopted to allow a hedger to express his/her view(s) in a flexible and realistic manner. The new model not only provides researchers with a conceptual framework to quantitatively analyze the impact on hedging positions of subjective views, but also provides practitioners with a tool to adjust hedging positions consistent with their views regarding future market price movements.

We illustrate the impact of subjective views on hedging decisions (and how to hedge selec-

tively) with four applications to an airline interested in hedging jet fuel price exposure. The numerical results confirm that subjective views can have a substantial impact on a hedger's optimal position. The impact is most evident when hedgers speculate on future market price directions and/or is pessimistic about the effectiveness of hedging, i.e., a breakdown in the correlation of different markets. With a moderate level of risk-aversion, the numerical results show that a directional view may change the optimal position from 34% to 170% of the "no view" position. By comparison, a view on the covariance matrix may change the optimal position from 83% to 102% of the "no view" position. The results offer new insight into the influence of subjective views on hedging behavior and contribute to explaining the large cross-sectional and time-series variation of hedging ratios often observed in practice.

References

Adam, T. R. and C. S. Fernando (2006). Hedging, speculation and shareholder value. *Journal of Financial Economics 81*(2), 283–309. forthcoming.

Anderson, R. W. and J.-P. Danthine (1980). Hedging and joint production: Theory and illustrations. *Journal of Finance 35*(2), 487–498.

Babcock, B. A., E. K. Choi, and E. Feinerman (1993). Risk and probability premiums for cara utility functions. *Journal of Agricultural and Resource Economics 18*(1), 17–24.

Barberis, N. and R. Thaler (2003). *Handbook of the Economics of Finance*, Chapter A Survey of Behavioral Finance, pp. 1053–1123. North Holland, Amsterdam.

Beber, A. and D. Fabbri (2005). Who times the foreign exchange market? corporate speculation and ceo characteristics. Working paper, University of Lausanne and FAME.

Bernardo, J. M. and A. F. M. Smith (1994). *Bayesian Theory*. Chichester; New York: Wiley.

Bevan, A. and K. Winkelmann (1998). Using the black-litterman global asset allocation model: Three years of practical experience. Technical report, Fixed Income Research, Goldman Sachs Group.

Black, F. and R. Litterman (1992). Global portfolio optimization. *Financial Analyst Journal 48*(5), 28–43.

Bodnar, G. M., G. S. Hayt, and R. C. Marston (1998). 1998 wharton survey of financial risk management by us non-financial firms. *Financial Management 27*(4), 70–91.

Brown, G. W., P. R. Crabb, and D. Haushalter (2006). Are firms successful at selective hedging? *Journal of Business 79*(2). forthcoming.

Brown, G. W. and Z. Khokher (2005). Corporate risk management and speculative motives. working paper, Kenan-Flagler Business School, University of North Carolina at Chapel Hill.

Carter, D., D. A. Rogers, and B. J. Simkins (2002). Does fuel hedging make economic sense? the case of the us airline industry. AFA 2004 San Diego Meetings. http://ssrn.com/abstract=325402.

Debondt, W. F. and R. H. Thaler (1995). *Handbooks in Operations Research and Management Science, 9: Finance*, Chapter Financial Decision-Making in Markets and Firms: A Behavioral Perspective, pp. 385–410. North Holland, Amsterdam.

Dolde, W. (1993). The trajectory of corporate financial risk management. *Journal of Applied Corporate Finance 6*(3), 33–41.

Fama, E. F. and K. H. French (1988). Business cycles and the behavior of metals prices. *Journal of Finance 43*(5), 1075–1093.

Fama, E. F. and K. R. French (1987). Commodity future prices: Some evidence on forecast power, premiums, and the theory of storage. *Journal of Business 60*(1), 55–73.

Faulkender, M. (2005). Hedging or market timing? selecting the interest rate exposure of corporate debt. *Journal of Finance 60*(2), 931–962.

French, K. R. (1986). Detecting spot price forecasts in futures prices. *Journal of Business 59*(2), 39–54.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1993). Risk management: Coordinating corporate investment and financing policies. *Journal of Finance 48*(5), 1629–1658.

Frost, P. A. and J. E. Savarino (1986). An empirical bayes approach to efficient portfolio selection. *Journal of Financial and Quantitative Analysis 21*(3), 293–305.

Fusal, G. and A. Meucci (2003). Assessing views. *Risk 16*(3), 18–21.

Garthwaite, P. H. and S. A. Al-Awadhi (2001). Non-conjugate prior distribution assessment for multivariate normal sampling. *Journal of the Royal Statistical Society: Series B (Statistical Methodology) 63*(1), 95–110.

Geczy, C., B. A. Minton, and C. M. Schrand (2005). Taking a view: Corporate speculation, governance and compensation. Working Paper, http://ssrn.com/abstract=633081.

Greene, W. H. (2002). *Econometric Analysis* (5th ed.). Upper Saddle River, N.J.: Prentice Hall.

Harvey, C. R., J. C. Liechty, M. W. Liechty, and P. Müller (2003). Portfolio selection with higher moments. working paper, Fuqua School of Business, Duke University.

He, G. and R. Litterman (1999). The intuition behind black-litterman model portfolios. Technical report, Quantitative Resources Group, Goldman Sachs Group, Inc.

Hull, J. C. (2002). *Options, Futures and Other Derivatives* (5 ed.). Upper Saddle River, NJ: Prentice Hall.

Jorion, P. (1986). Bayes-stein estimator for portfolio analysis. *Journal of Financial and Quantitative Analysis 21*(3), 279–292.

Lence, S. H. and D. J. Hayes (1994). Parameter-based decision markting under estimation risk: An application to futures trading. *Journal of Finance 49*(1), 345–357.

Myers, R. J. and S. R. Thompson (1989). Generalized optimal hedge ratio estimation. *American Journal of Agricultural Economics 71*(4), 858–868.

Ng, V. K. and S. C. Pirrong (1994). Fundamentals and volatility: Storage, spreads, and the dynamics of metals prices. *Journal of Business 67*(2), 203–230.

O'Hagan, A., J. Forster, M. G. Kendall, A. Stuart, and J. K. Ord (2004). *Kendall's Advanced Theory of Statistics: Bayesian Inference*. London: Arnold.

Pástor, v. (2000). Portfolio selection and asset pricing models. *Journal of Finance 55*(1), 179–223.

Scherer, B. (2004). *Portfolio Construction and Risk Budgeting*. London: Risk Books.

Shi, W. and S. H. Irwin (2005). Optimal hedging with a subjective view: An empirical bayesian approach. *American Journal of Agricultural Economics 87*(4), 918–930.

Shiller, R. J. (2003). From efficient markets theory to behavioral finance. *Journal of Economic Perspectives 17*(1), 83–104.

Smith, C. W. and R. M. Stulz (1985). The determinants of firms' hedging policies. *Journal of Financial and Quantitative Analysis 20*(4), 391–405.

Stulz, R. M. (1984). Optimal hedging policies. *Journal of Financial and Quantitative Analysis 19*(2), 127–140.

Stulz, R. M. (1996). Rethinking risk management. *Journal of Applied Corporate Finance 9*(3), 8–24.

Tufano, P. (1996). Who manages risk? an empirical examination of risk management practices in the gold mining industry. *Journal of Finance 51*(4), 1097–1137.

Tufano, P. (1998). The determinants of stock price exposure: Financial engineering and the gold mining industry. *Journal of Finance 53*(3), 1015–1052.

Williams, J. C. and B. D. Wright. (1991). *Storage and Commodity Markets*. Cambridge; New York: Cambridge University Press.

Working, H. (1962). New concepts concerning futures markets and prices. *American Economic Review 52*(3), 432–459.

Tables and Figures

Table 1: A Hedger's Optimal Position With a Directional View.

Subjective View	Confidence Level	Weight of Evidence	Speculative Component	Hedging Component	Optimal Position
Panel A: Risk Premium=50%					
No view	NA	1.00	-32,440	1,560,187	1,527,746
Bullish	High	0.80	413,071	1,560,187	1,973,258
	Low	0.97	246,004	1,560,187	1,806,191
Bearish	High	0.84	-451,999	1,560,187	1,108,187
	Low	0.98	-294,665	1,560,187	1,265,522
Panel B: Risk Premium=25%					
No view	NA	1.00	-75,722	1,560,187	1,484,464
Bullish	High	0.80	964,198	1,560,187	2,524,384
	Low	0.97	574,228	1,560,187	2,134,414
Bearish	High	0.84	-1,055,065	1,560,187	505,122
	Low	0.98	-687,811	1,560,187	872,375
Panel C: Risk Premium=10%					
No view	NA	1.00	-196,362	1,560,187	1,363,825
Bullish	High	0.80	2,500,337	1,560,187	4,060,524
	Low	0.97	1,489,075	1,560,187	3,049,262
Bearish	High	0.84	-2,735,971	1,560,187	-1,175,785
	Low	0.98	-1,783,618	1,560,187	-223,431

The hedger is assumed to have either a bullish or bearish view regarding the expected futures price change. A bullish (bearish) view means that the hedger speculates the futures price change will be \$0.03/gallon (\$ − 0.03/gallon), rather than \$ − 0.0009/gallon according to the market consensus. A directional view is associated with either a high ($\kappa_1 = 10$) or a low confidence level ($\kappa_1 = 5$). In addition, the hedger is assumed to have no view regarding the covariance matrix, signified by $\nu_1 = 0$. The prior is calibrated with historical data and the confidence levels associated the prior are set as $\kappa_0 = 15$ and $\nu_0 = 15$. Speculative, hedging and overall optimal positions are displayed with gallons as the unit of measurement.

Table 2: A Hedger's Optimal Position with a View on the Covariance Matrix.

Subjective View		Confidence	Weight of	Speculative	Hedging	Optimal
Volatilities	Correlation	Level	Evidence	Component	Component	Position
Panel A: Risk Premium=50%						
No view	No view	NA	1.00	-32,440	1,560,187	1,527,746
No view	Decreasing	High	0.55	-38,112	1,404,168	1,366,056
		Low	0.82	-35,951	1,462,675	1,426,724
Increasing	No view	High	0.20	-17,323	1,560,187	1,542,863
		Low	0.56	-20,543	1,560,187	1,539,643
Increasing	Decreasing	High	0.00	-17,323	1,276,516	1,259,193
		Low	0.02	-20,543	1,337,303	1,316,759
Panel B: Risk Premium=25%						
No view	No view	NA	1.00	-75,722	1,560,187	1,484,464
No view	Decreasing	High	0.55	-88,961	1,404,168	1,315,207
		Low	0.82	-83,917	1,462,675	1,378,758
Increasing	No view	High	0.20	-40,437	1,560,187	1,519,750
		Low	0.56	-47,953	1,560,187	1,512,234
Increasing	Decreasing	High	0.00	-40,437	1,276,516	1,236,080
		Low	0.02	-47,953	1,337,303	1,289,350
Panel C: Risk Premium=10%						
No view	No view	NA	1.00	-196,362	1,560,187	1,363,825
No view	Decreasing	High	0.55	-230,691	1,404,168	1,173,477
		Low	0.82	-217,613	1,462,675	1,245,062
Increasing	No view	High	0.20	-104,859	1,560,187	1,455,327
		Low	0.56	-124,350	1,560,187	1,435,836
Increasing	Decreasing	High	0.00	-104,859	1,276,516	1,171,657
		Low	0.02	-124,350	1,337,303	1,212,953

The hedger is assumed to be pessimistic about the hedging effectiveness and suspect that the volatilities of futures and spot markets will be twice that of the prior, and/or the correlation between the two markets could be three-quarters that of the prior. A view on the covariance matrix is associated with either a high ($v_1 = 10$) or a low confidence level ($v_1 = 5$). The hedger is assumed to have no view regarding the expectation vector, signified by $\kappa_1 = 0$. The prior is calibrated with historical data and the confidence levels associated the prior are set as $\kappa_0 = 15$ and $v_0 = 15$. Speculative, hedging and overall optimal positions are displayed with gallons as the unit of measurement.

Table 3: A Hedger's Optimal Position with a Combined View.

Combined View			Confidence Level	Weight of Evidence	Speculative Component	Hedging Component	Optimal Position
Expectation	Volatility	Correlation					
			Risk Premium=25%				
No view	No view	No view	NA	1.00	-75,722	1,560,187	1,484,464
No view	No view	Decreasing	High	0.55	-88,961	1,404,168	1,315,207
			Low	0.82	-83,917	1,462,675	1,378,758
No view	Increasing	No view	High	0.20	-40,437	1,560,187	1,519,750
			Low	0.56	-47,953	1,560,187	1,512,234
No view	Increasing	Decreasing	High	0.00	-40,437	1,276,516	1,236,080
			Low	0.02	-47,953	1,337,303	1,289,350
Bullish	No view	No view	High	0.80	964,198	1,560,187	2,524,384
			Low	0.97	574,228	1,560,187	2,134,414
Bullish	No view	Decreasing	High	0.27	1,132,764	1,404,168	2,536,932
			Low	0.68	636,374	1,462,675	2,099,049
Bullish	Increasing	No view	High	0.14	514,893	1,560,187	2,075,079
			Low	0.49	363,642	1,560,187	1,923,829
Bullish	Increasing	Decreasing	High	0.00	514,893	1,276,516	1,791,409
			Low	0.01	363,642	1,337,303	1,700,945
Bearish	No view	No view	High	0.84	-1,055,065	1,560,187	505,122
			Low	0.98	-687,811	1,560,187	872,375
Bearish	No view	Decreasing	High	0.29	-1,239,516	1,404,168	164,651
			Low	0.70	-762,250	1,462,675	700,425
Bearish	Increasing	No view	High	0.14	-563,417	1,560,187	996,770
			Low	0.49	-435,571	1,560,187	1,124,615
Bearish	Increasing	Decreasing	High	0.00	-563,417	1,276,516	713,100
			Low	0.01	-435,571	1,337,303	901,732

The hedger is assumed to have a "combined" view that is a combination of a directional view and a view on the covariance matrix. A bullish (bearish) directional view means that the hedger speculates the futures price change will be $0.03/gallon ($ − 0.03/gallon), rather than $ − 0.0009/gallon according to the market consensus. A covariance matrix view means that the hedger suspects that the volatilities of futures and spot markets will be twice that of the prior, and/or the correlation between the two markets could be three-quarters that of the prior. A directional view is associated with either a high ($\kappa_1 = 10$) or a low confidence level ($\kappa_1 = 5$). A view on the covariance matrix is associated with either a high ($\nu_1 = 10$) or a low confidence level ($\nu_1 = 5$). If the hedger expresses views on both the expectation vector and the covariance matrix, we restrict confidence levels to be the same for both views. The prior is calibrated with historical data and the confidence levels associated the prior are set as $\kappa_0 = 15$ and $\nu_0 = 15$. Speculative, hedging and overall optimal positions are displayed with gallons as the unit of measurement.

Table 4: A Hedger's Optimal Position With a Basis View.

Subjective View	Confidence Level	Weight of Evidence	Speculative Component	Hedging Component	Optimal Position
Panel A: Risk Premium=50%					
No view	NA	1.00	-32,440	1,560,187	1,527,746
Stronger	High	0.00	-223,968	1,276,516	1,052,548
	Low	0.01	-173,703	1,337,303	1,163,600
Weaker	High	0.02	755,549	1,560,187	2,315,736
	Low	0.46	367,178	1,560,187	1,927,365
Panel B: Risk Premium=25%					
No view	NA	1.00	-75,722	1,560,187	1,484,464
Stronger	High	0.00	-522,790	1,276,516	753,726
	Low	0.01	-405,460	1,337,303	931,843
Weaker	High	0.14	1,763,615	1,560,187	3,323,802
	Low	0.57	857,073	1,560,187	2,417,260
Panel C: Risk Premium=10%					
No view	NA	1.00	-196,362	1,560,187	1,363,825
Stronger	High	0.00	-1,355,688	1,276,516	-79,172
	Low	0.01	-1,051,430	1,337,303	285,873
Weaker	High	0.02	4,573,369	1,560,187	6,133,555
	Low	0.46	2,222,544	1,560,187	3,782,731

The hedger is assumed to have either a stronger or weaker basis view. The market consensus is that the basis will be strengthened by \$0.0011/gallon in a month, but the hedger with a stronger basis view expects the basis will be \$0.02/gallon, together with volatilities of spot and futures increasing by 100%, and the correlation dropping by 25% w.r.t. the market consensus. On the other hand, the hedger with a weaker basis view expects that the basis will be $\$-0.02$/gallon, together with volatilities of spot and futures decreasing by 50% and the correlation remaining the same w.r.t. the market consensus. A basis view is associated with either a high ($\kappa_1 = 10$ and $\nu_1 = 10$) or a low confidence level ($\kappa_1 = 5$ and $\nu_1 = 5$). The prior is calibrated with historical data and the confidence levels associated the prior are set as $\kappa_0 = 15$ and $\nu_0 = 15$. Speculative, hedging and overall optimal positions are displayed with gallons as the unit of measurement.